MCMSS-20165

[99.007]

E.V.Z.

Aug-1850

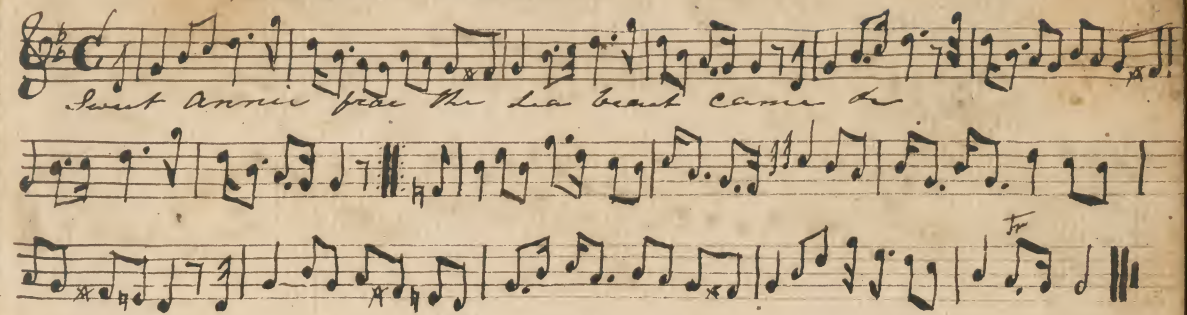
Buchanan's Journal of Man is devoted to the Science of Anthropology and is published at Cincinnati

Book of Milton Barlow
born abt 1784. (See his faint signature at top of front board)

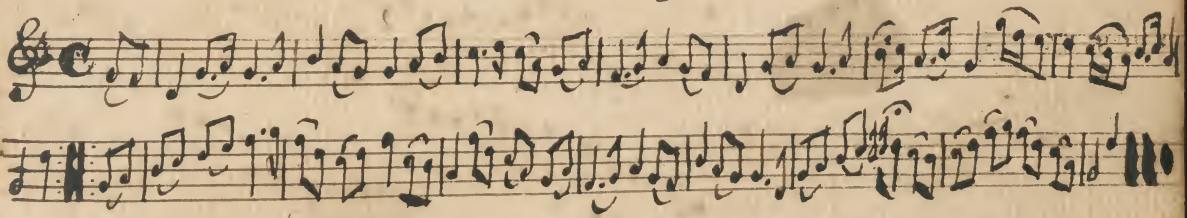
U KK
465

bought of a Chestnut Hill Mass dealer
RHF

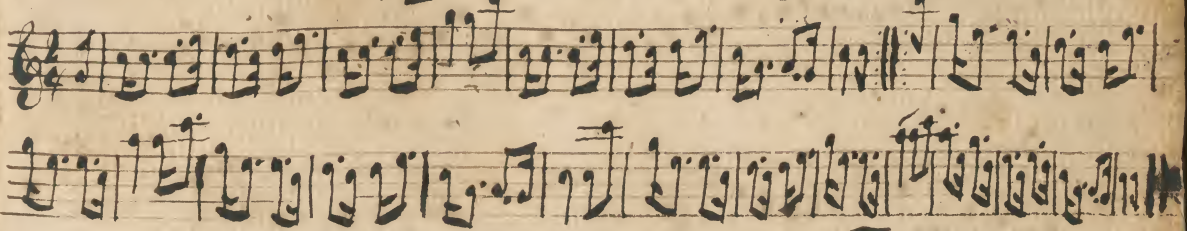
Sweet Annie



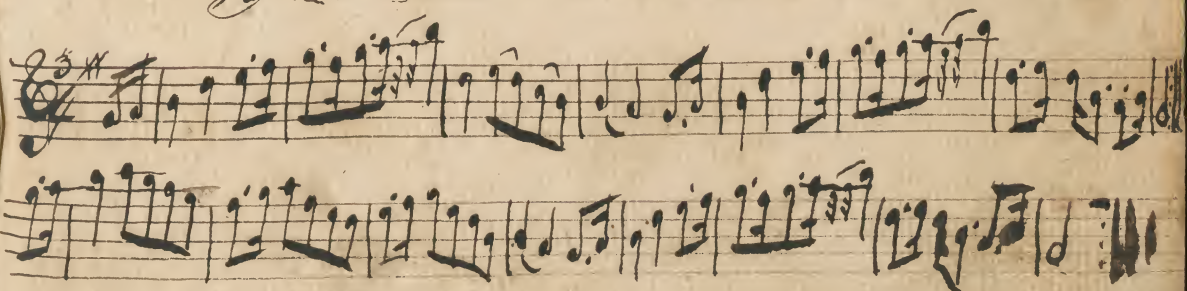
Highland Mary



Coming thro the Rye



Yellow Haired Laddie



[redacted]

Boy's wife.

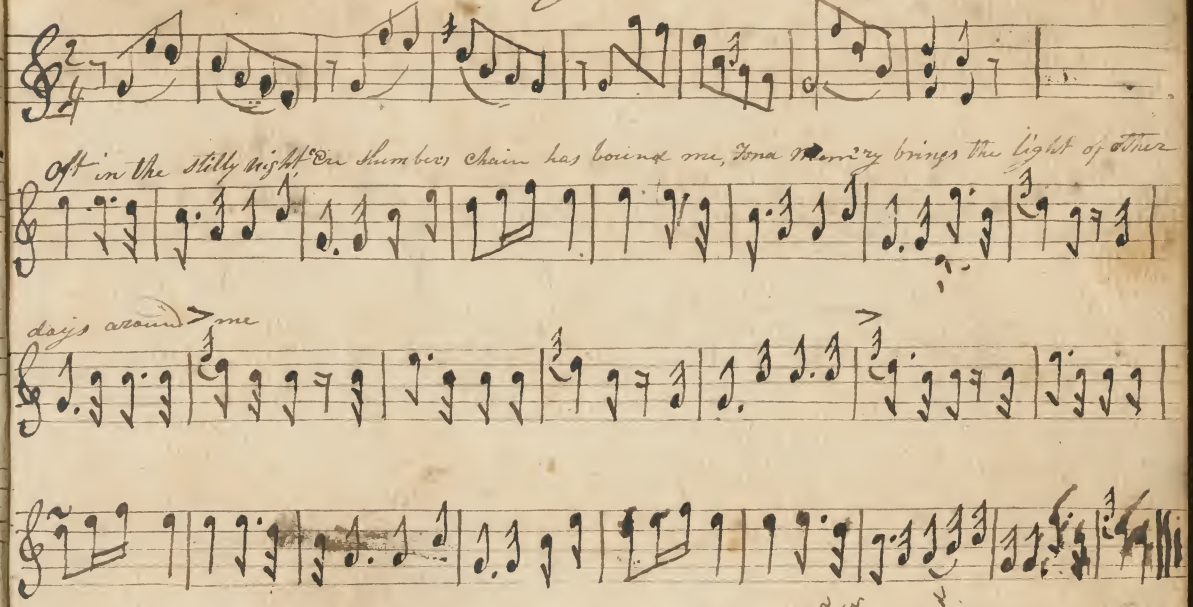
D.C.

Port Gordon.

Meeting of the Waters.

How blithe have I been wi' my Sandy

Off in the Stilly Night



Off in the Stilly Night.
Ere Slumbers Chain has bound me
Fond Memory brings the light
Of other days around me
The Smiles the tears
Of Boyhoods years
The words of love then spoken
The eyes that shone
Now dimmed & gone
The cheerful hearts now broken
Thus in the Stilly night, Ere Slumbers chain has bound me
Sad Memory brings the light, Of other days around me

When I remember all, the friends so linked together
I've seen around me fall, Like leaves in wintry weather,
I feel like one, Who treads alone
Some Banquet Hall deserted
Whose Lights are fled, Whose garlands dead
And all but he departed —

Thus in the Stilly night
Ere Slumbers chain has bound me
Sad Memory brings the light
Of other days around me

Jessie the

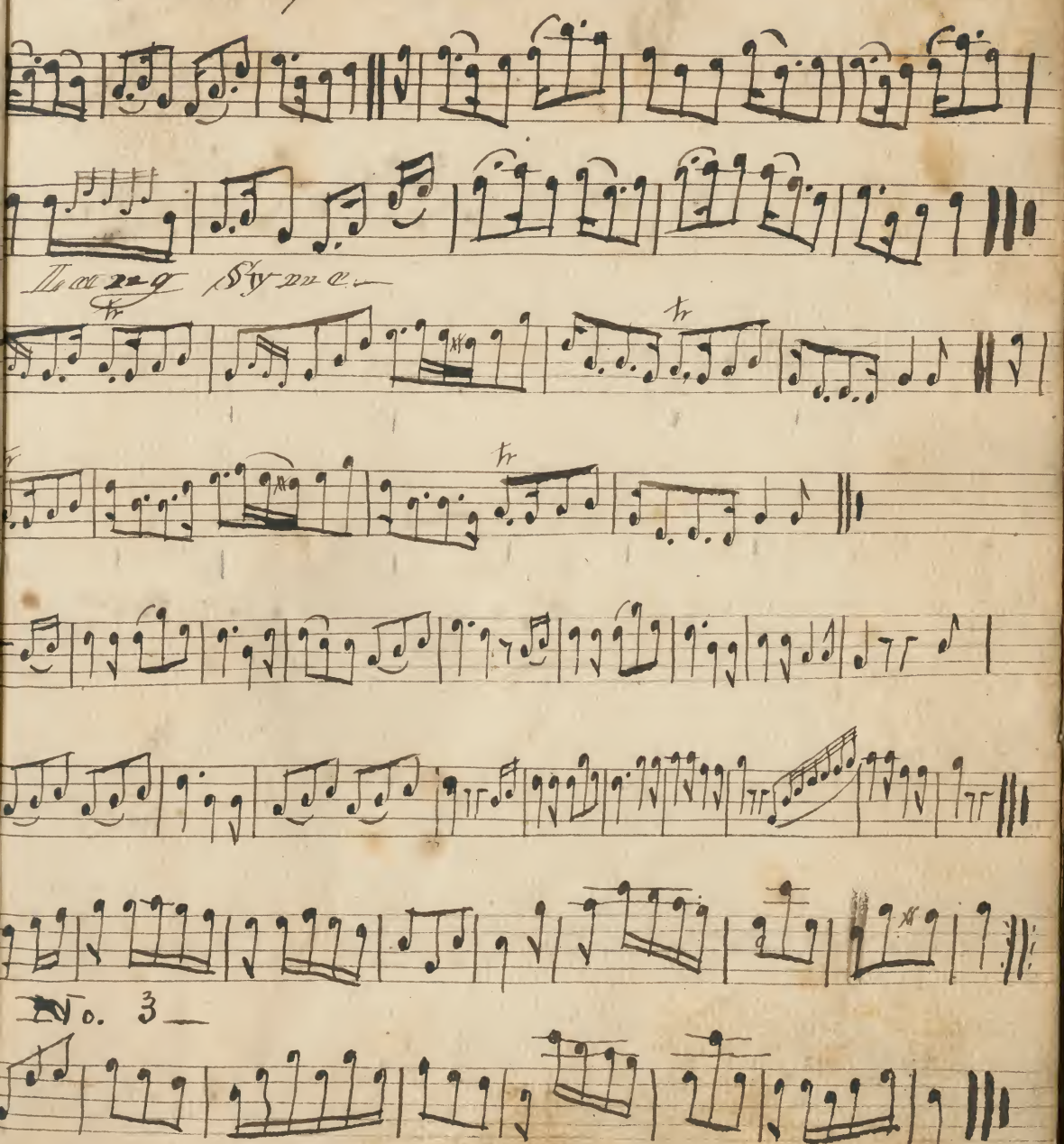
Auld

Blue Eyed Mary

Presto.

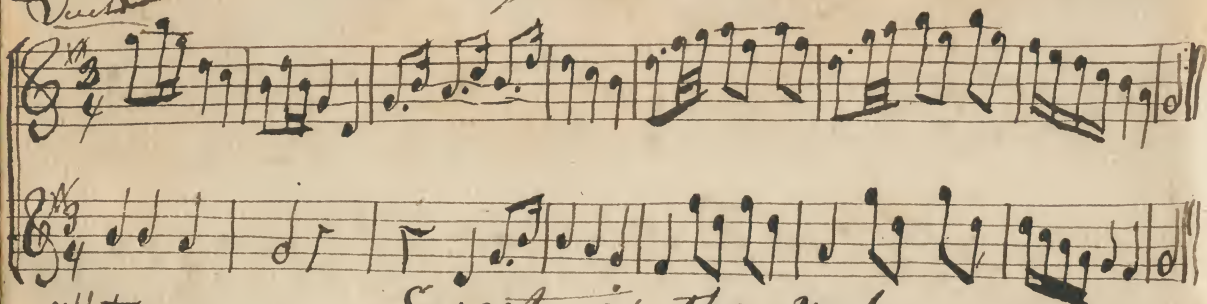
Waltz No 3

Waltz


Harmony
No. 3

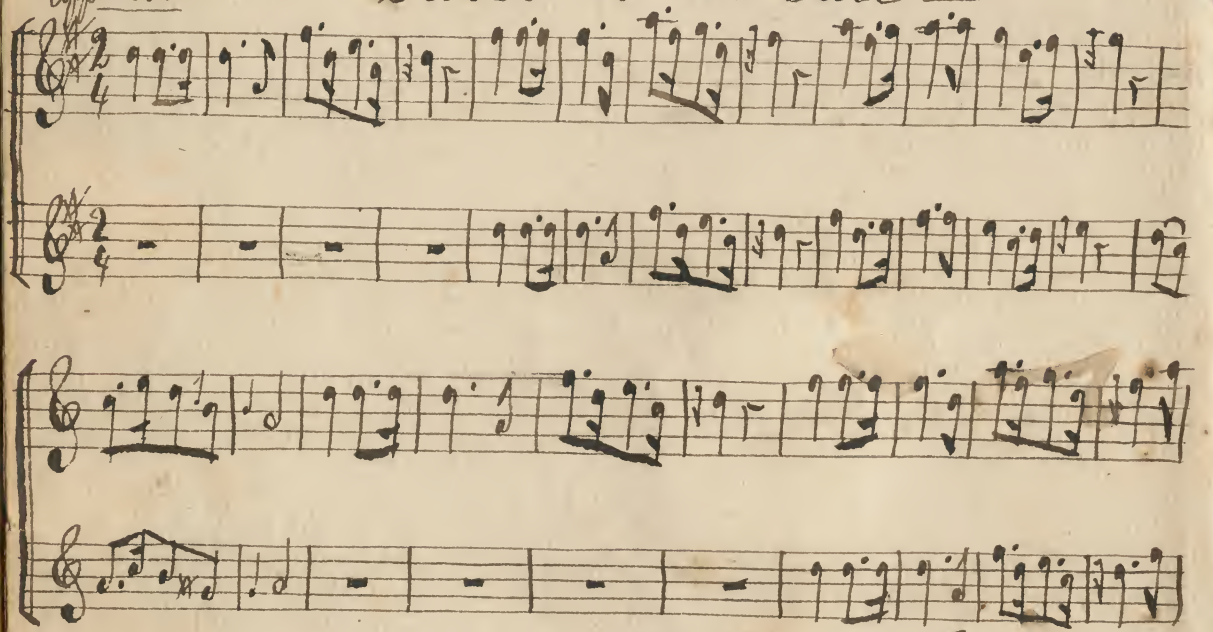
Ambassador's Minuet.

Duett

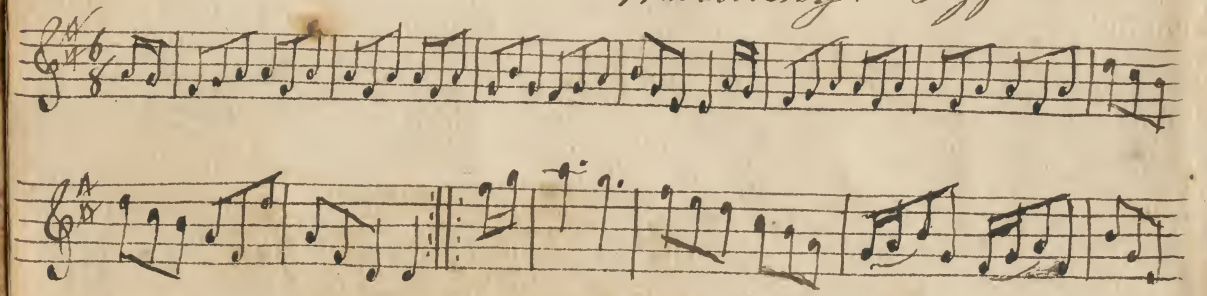


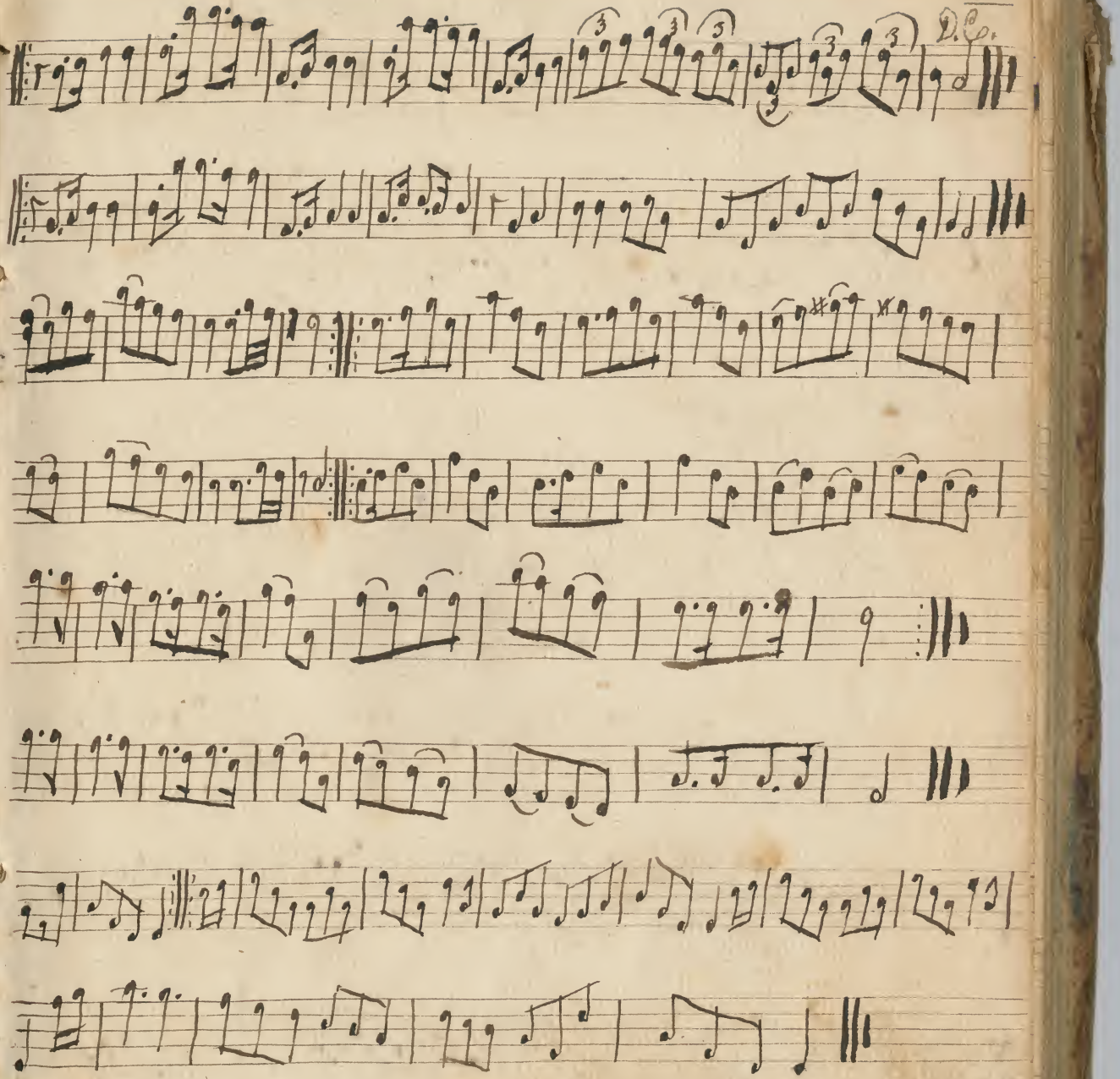
Sweet is the vale —

Affetuoso

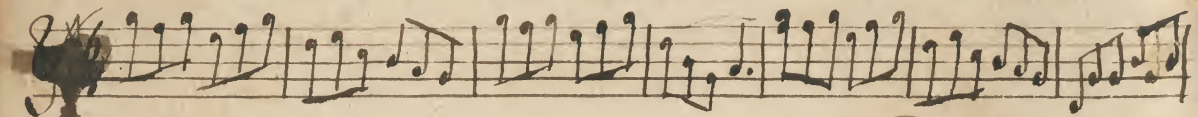


Mollonny's Jigg —

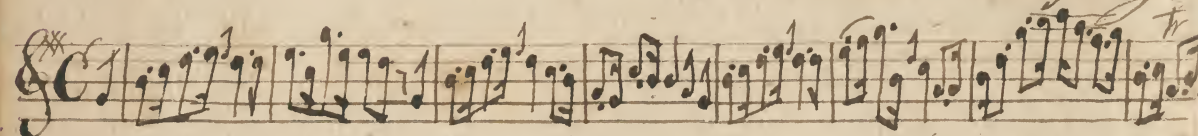



D.C.

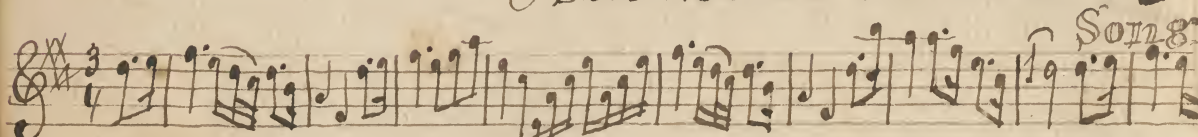
Jac Kson's Welcom



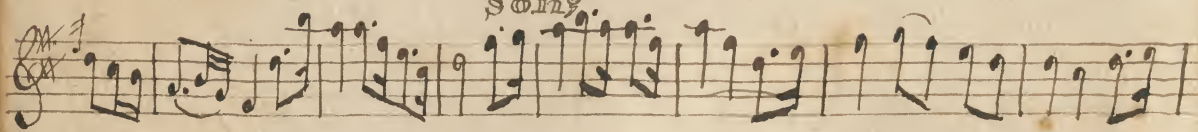
Auld Robin Gray



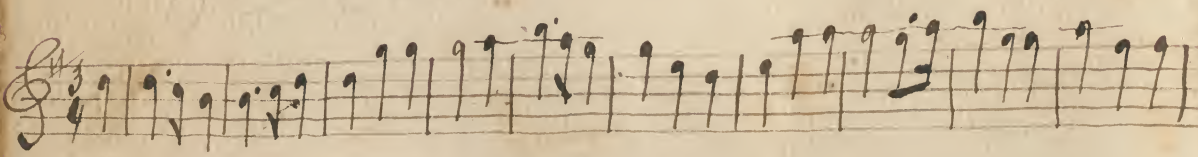
Durandarte & Belerma



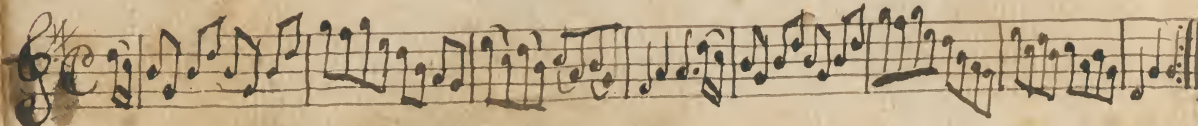
Song.



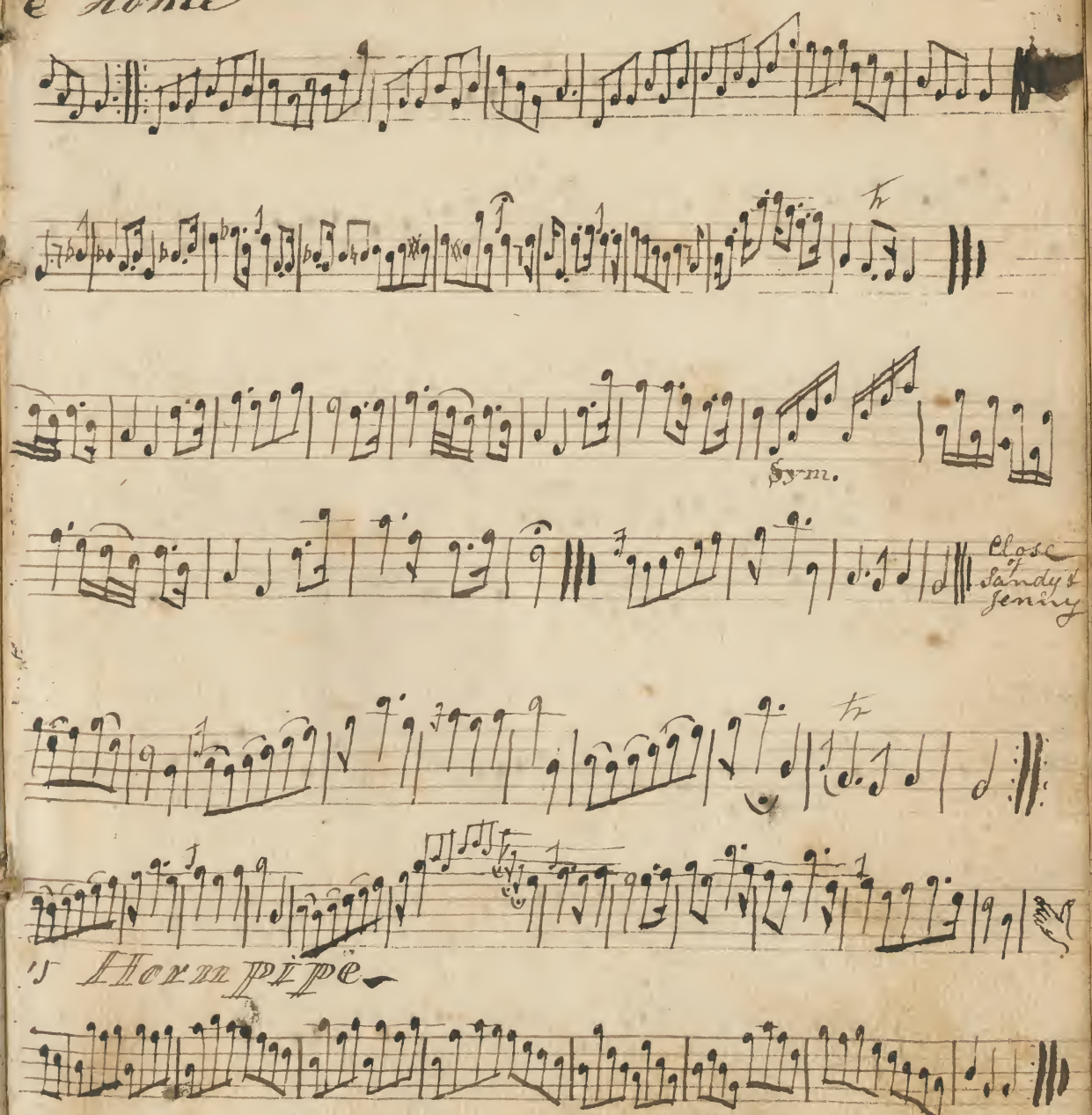
Sandy & Jenny



Mills Allan

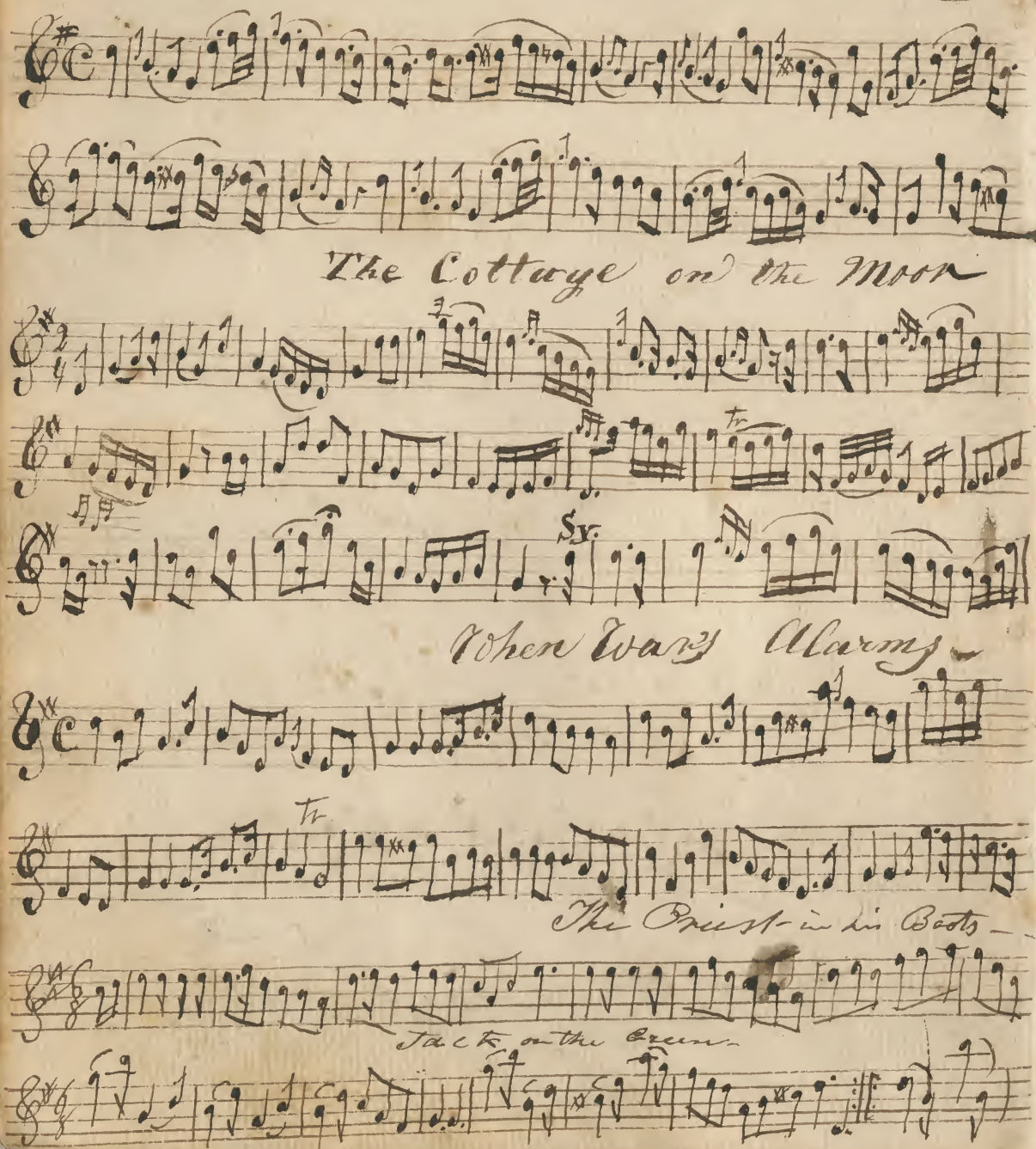


e Home

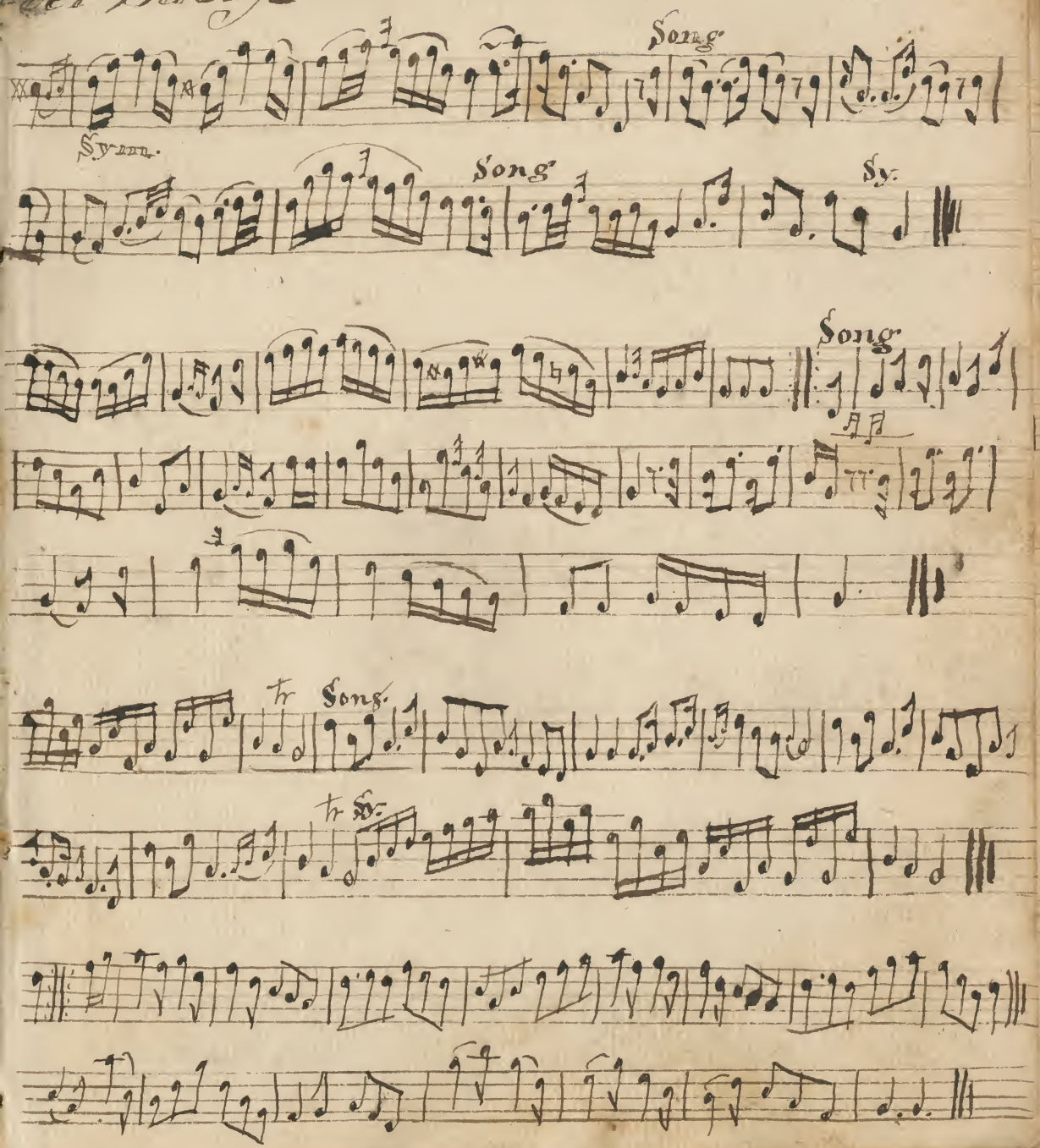


's Hornpipe

Shepherd's Dance

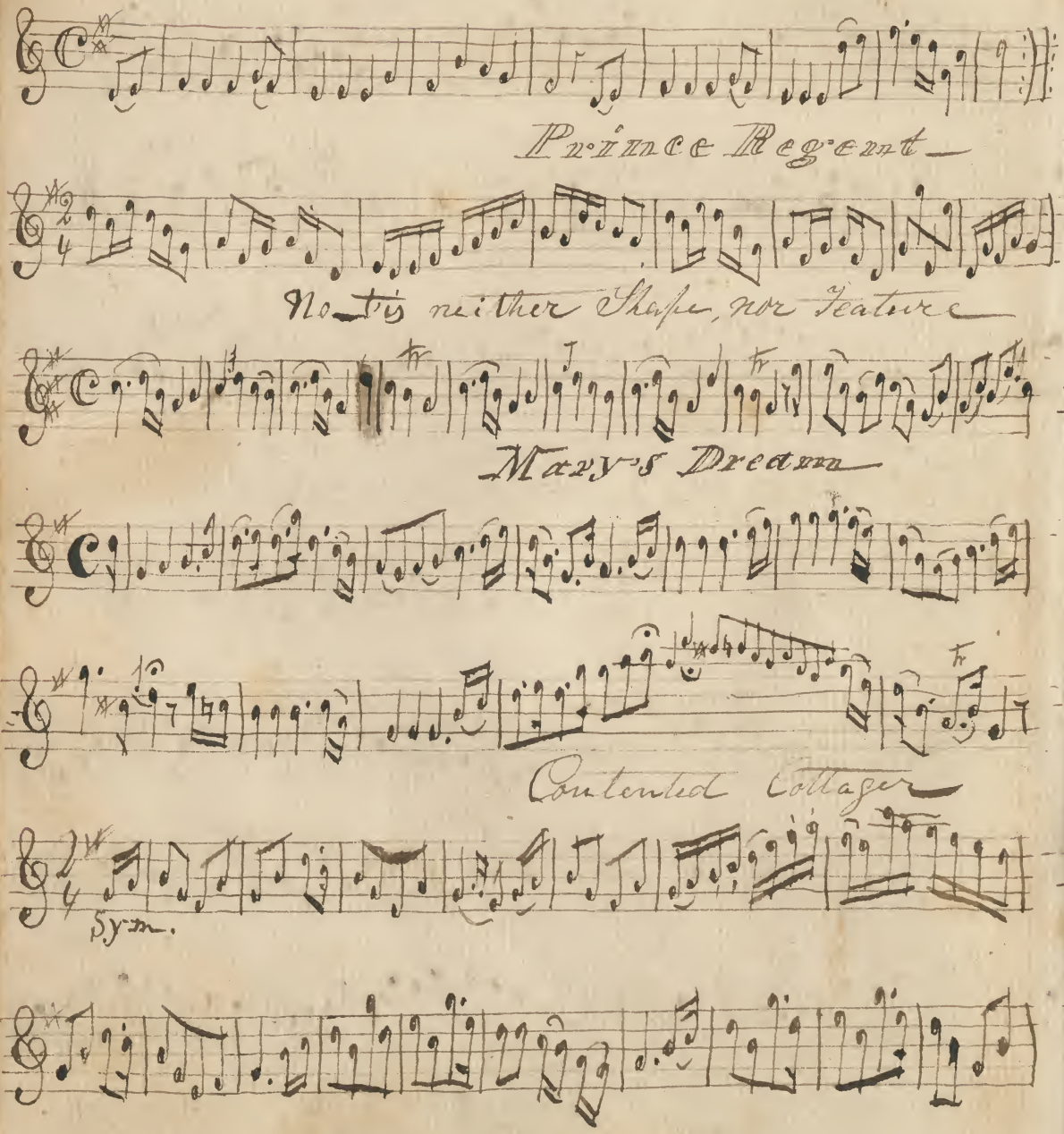


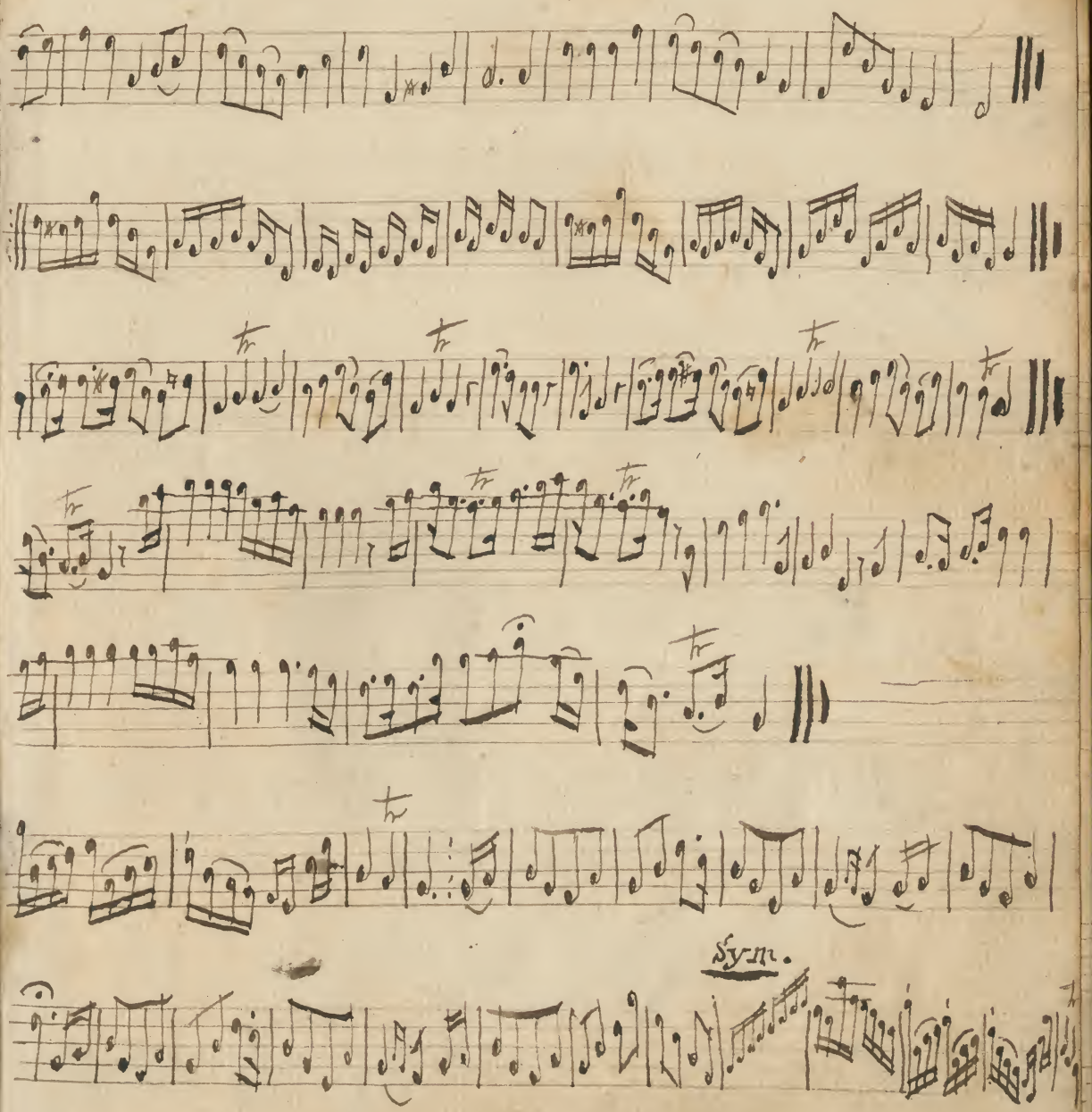
ter Sally



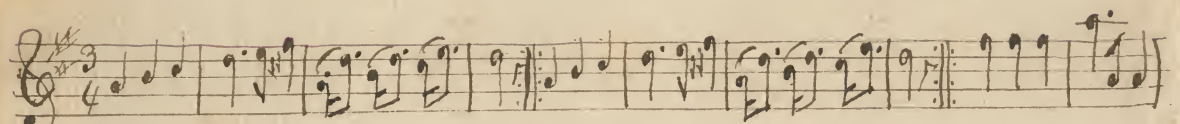
11/12

Blodget's Hymn

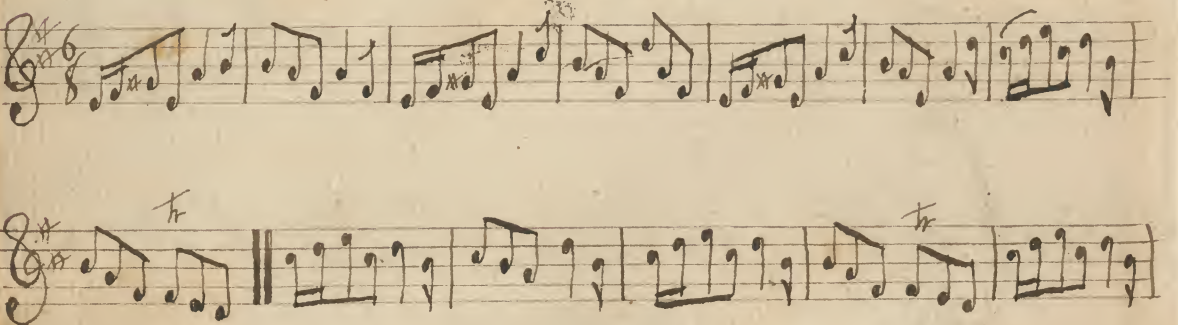



Syn.

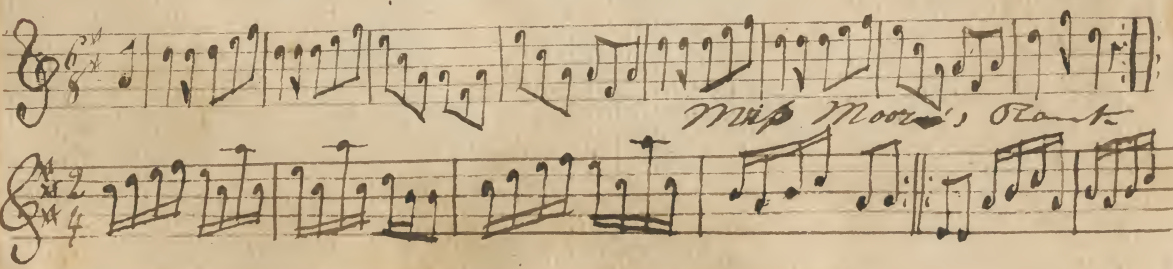
Robin Adier.

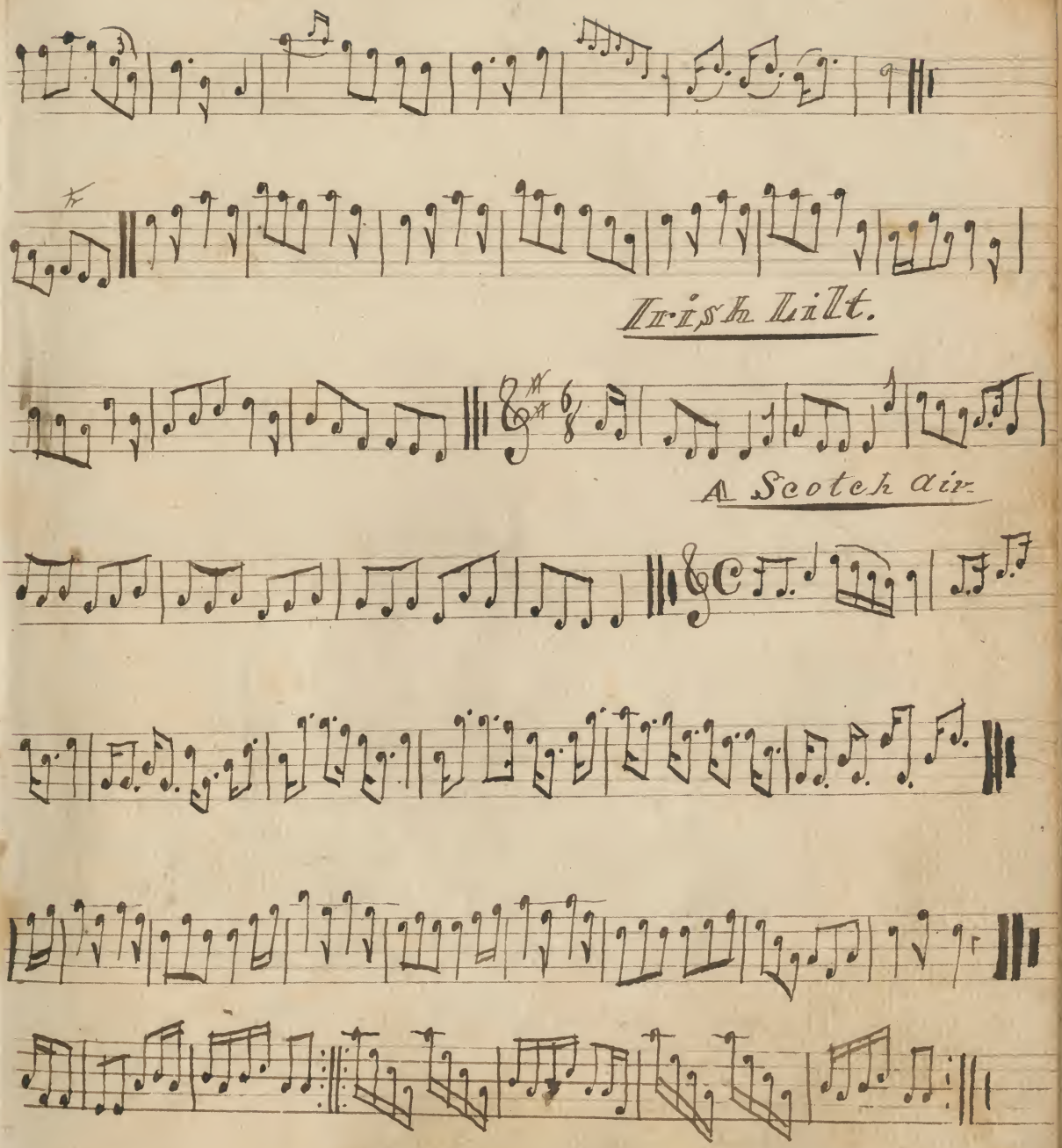


Humours of Panteen.

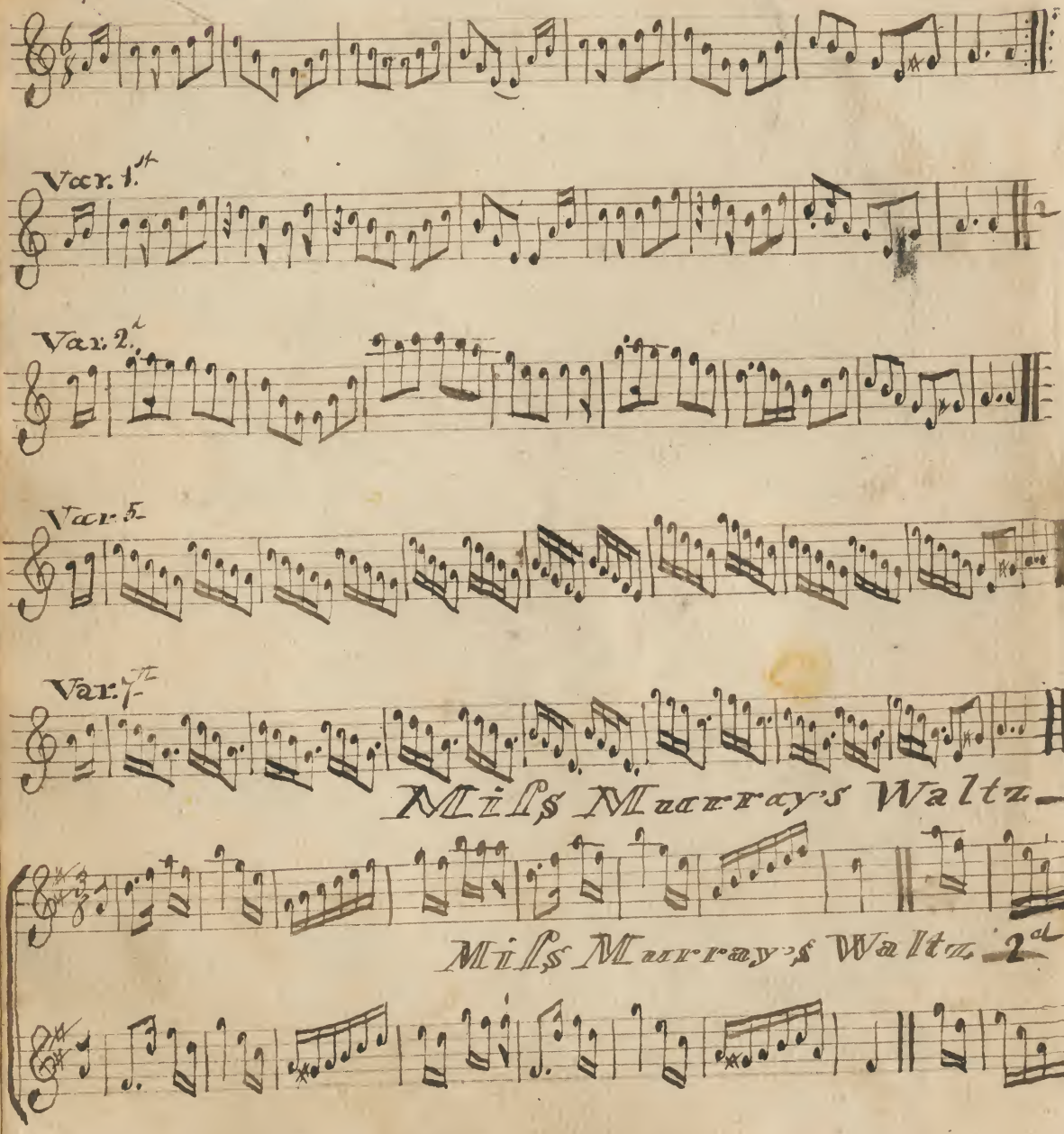


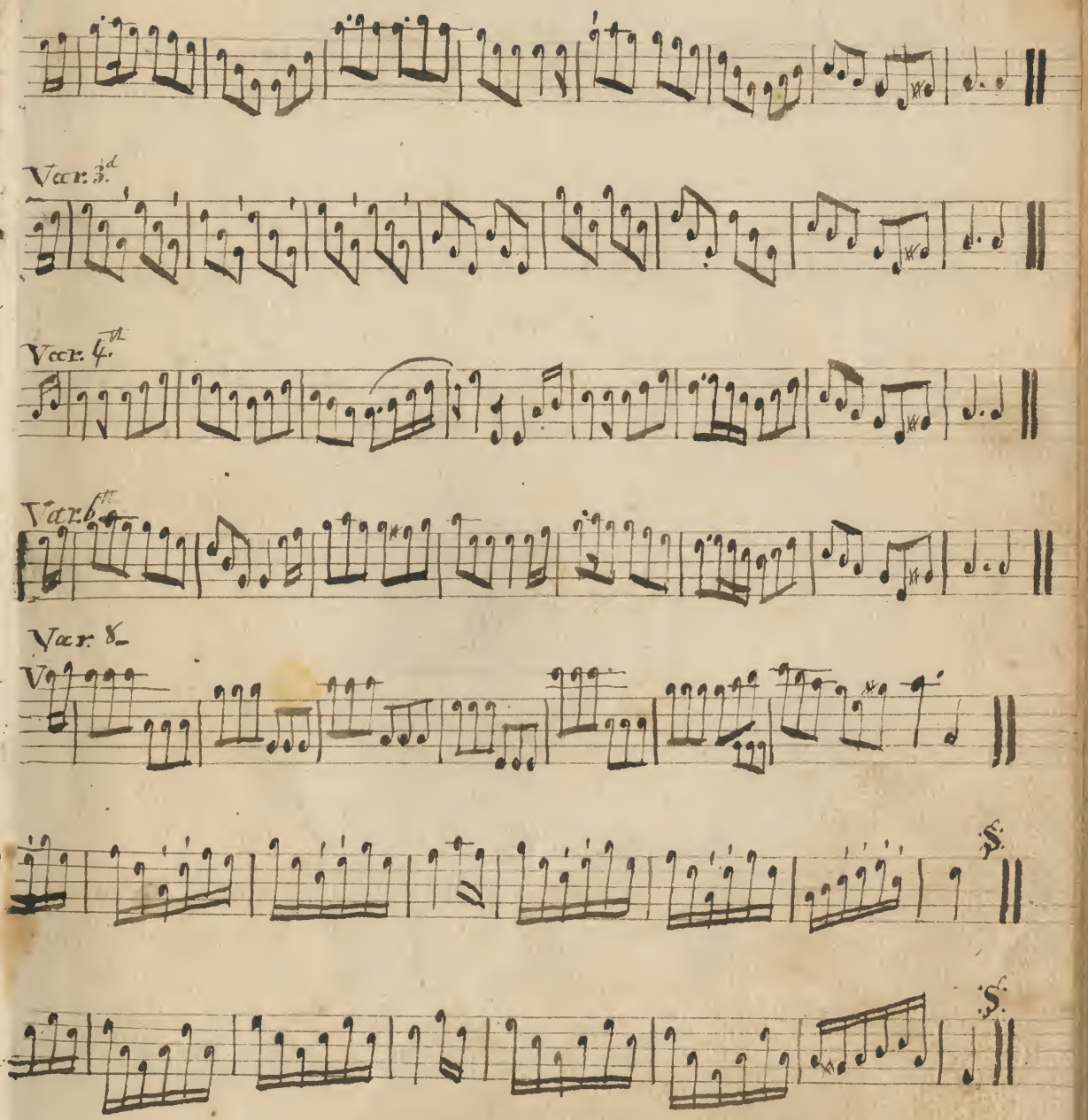
The New Rigg'd Ship.



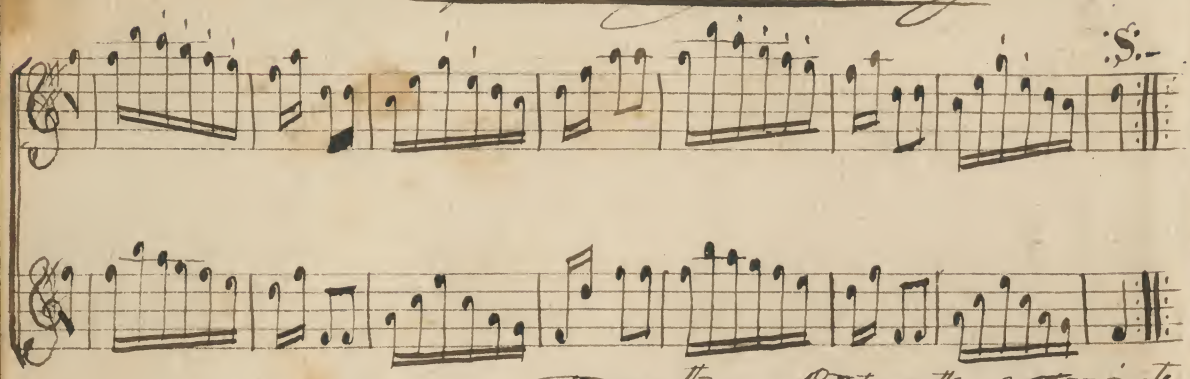

Irish Lilt.
A Scotch air

Green Sleeves, with variations.

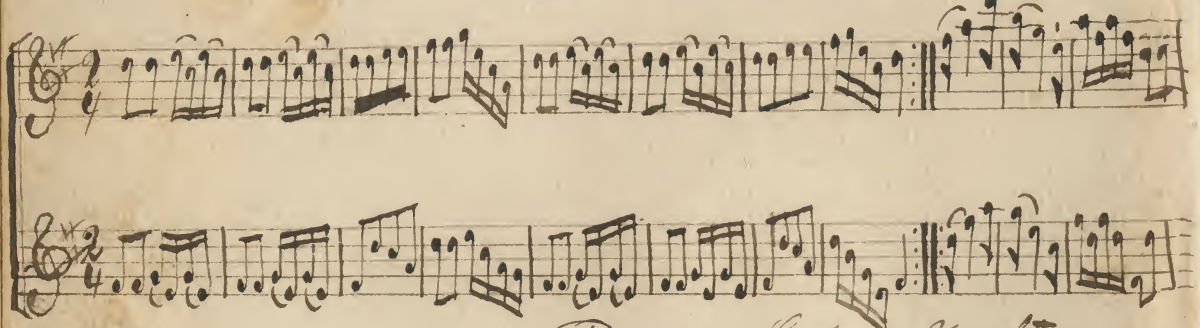



Var. 3.d
Var. 4.th
Var. 5.th
Var. 8.
Var.
S.
S.

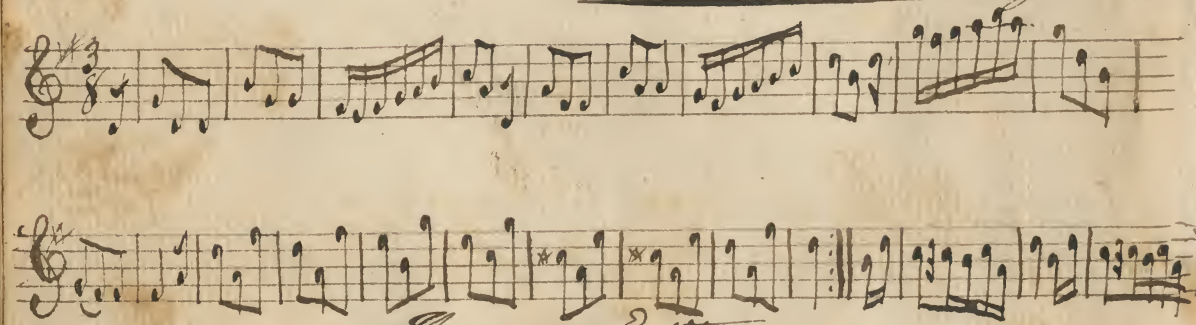
Miss Murray's Waltz



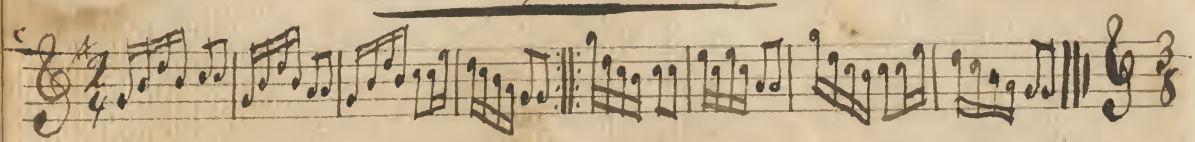
Duke of York's quick



Prince Leopold's Waltz



The vex'd Editor



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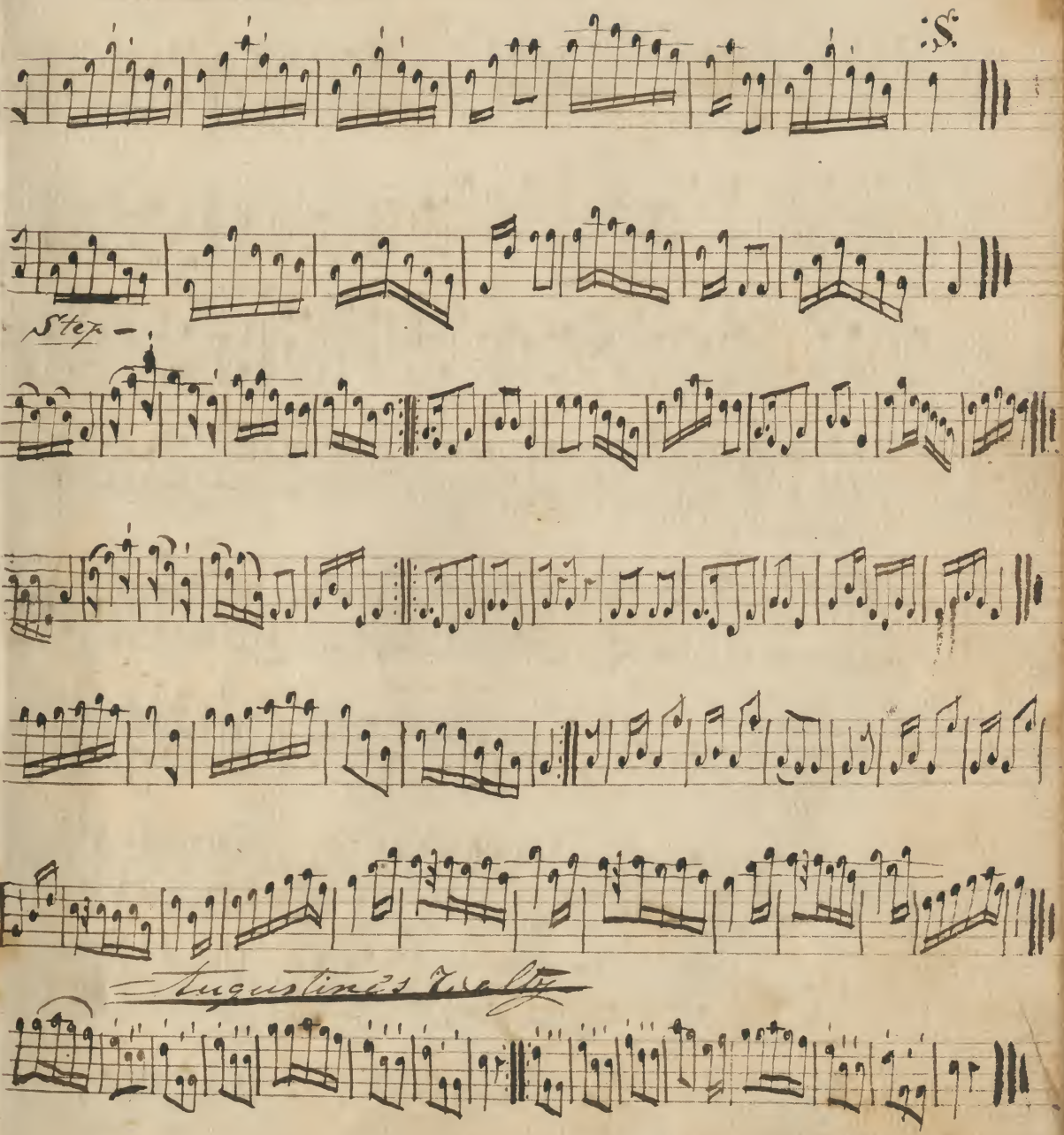


20

Yankee Doodle. with Variations

First

Second

Var. 2.

S. Var. 3.

S. Var. 4.

D.C. Var. 1.

Var. 9.

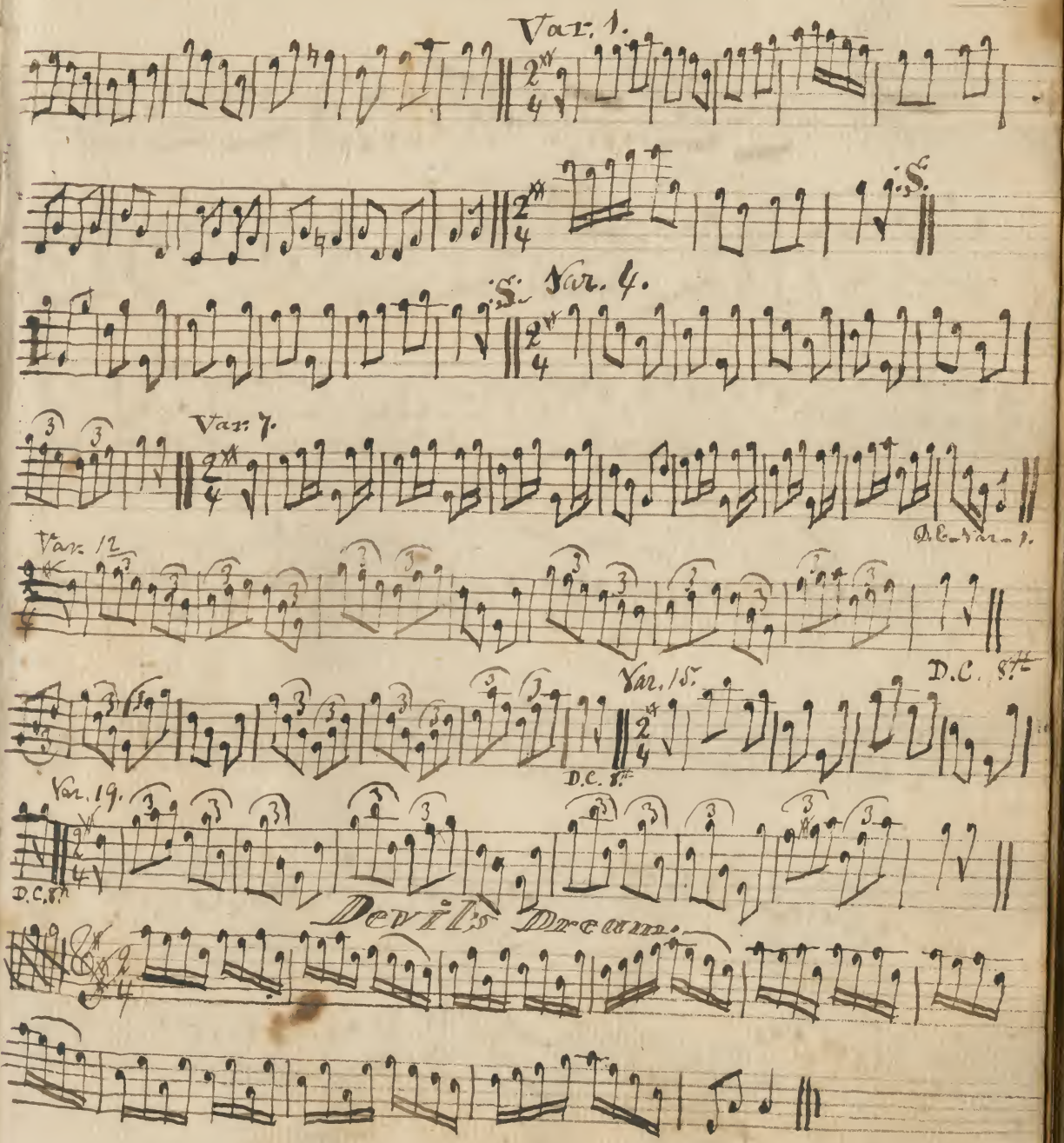
Var. 13.

Var. 14.

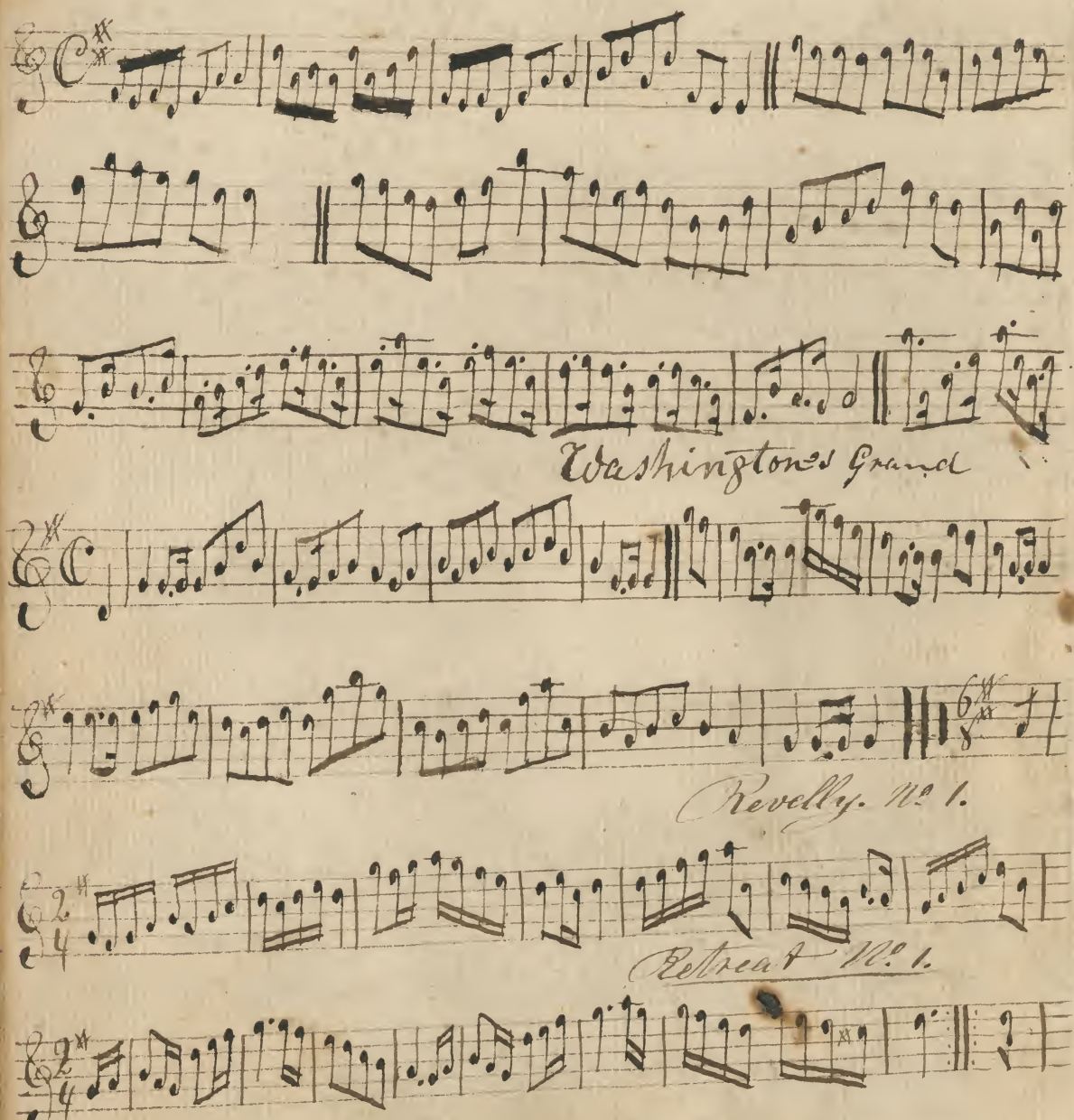
D.C. 8.

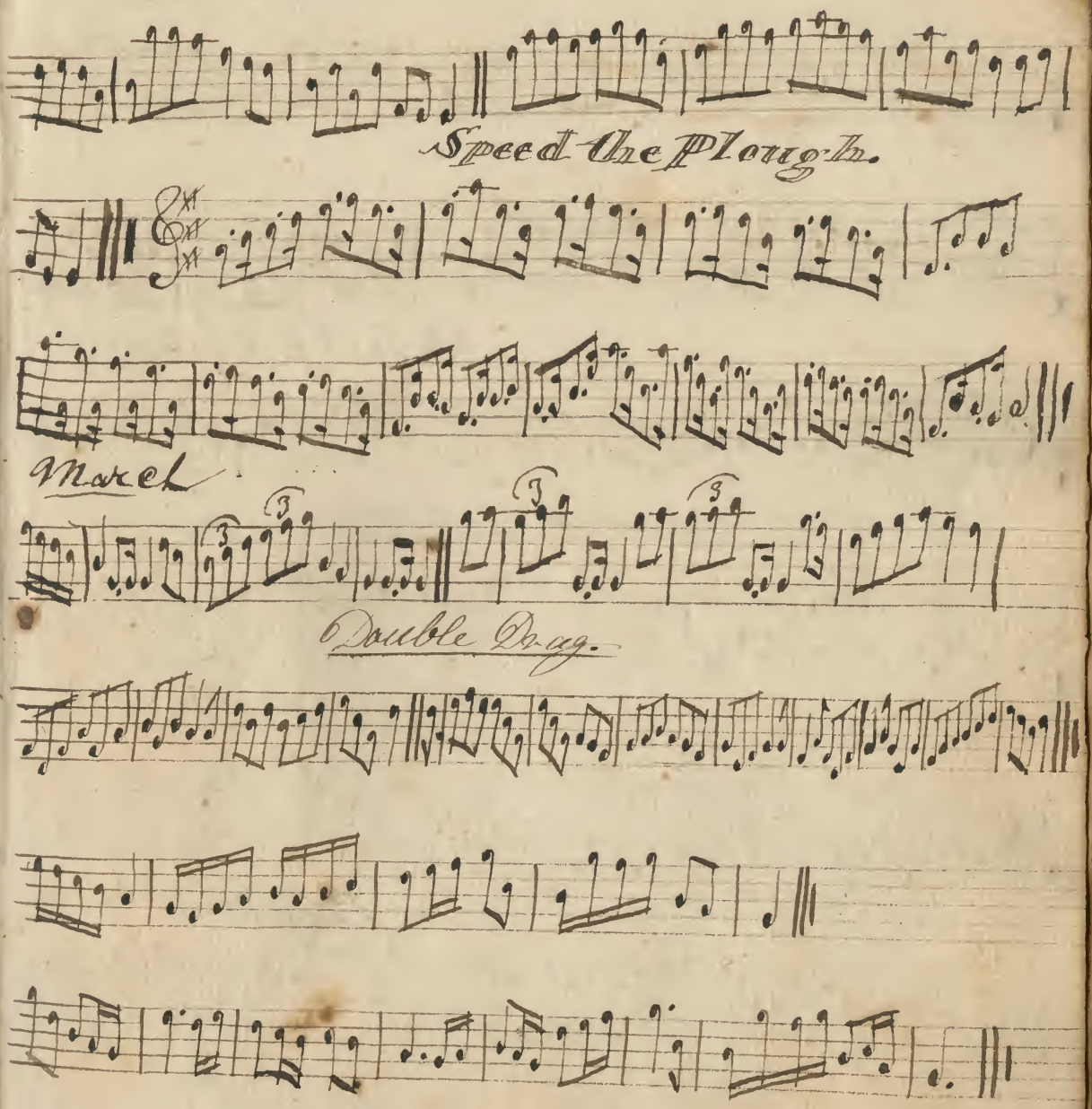
Var. 18.

Var. 20

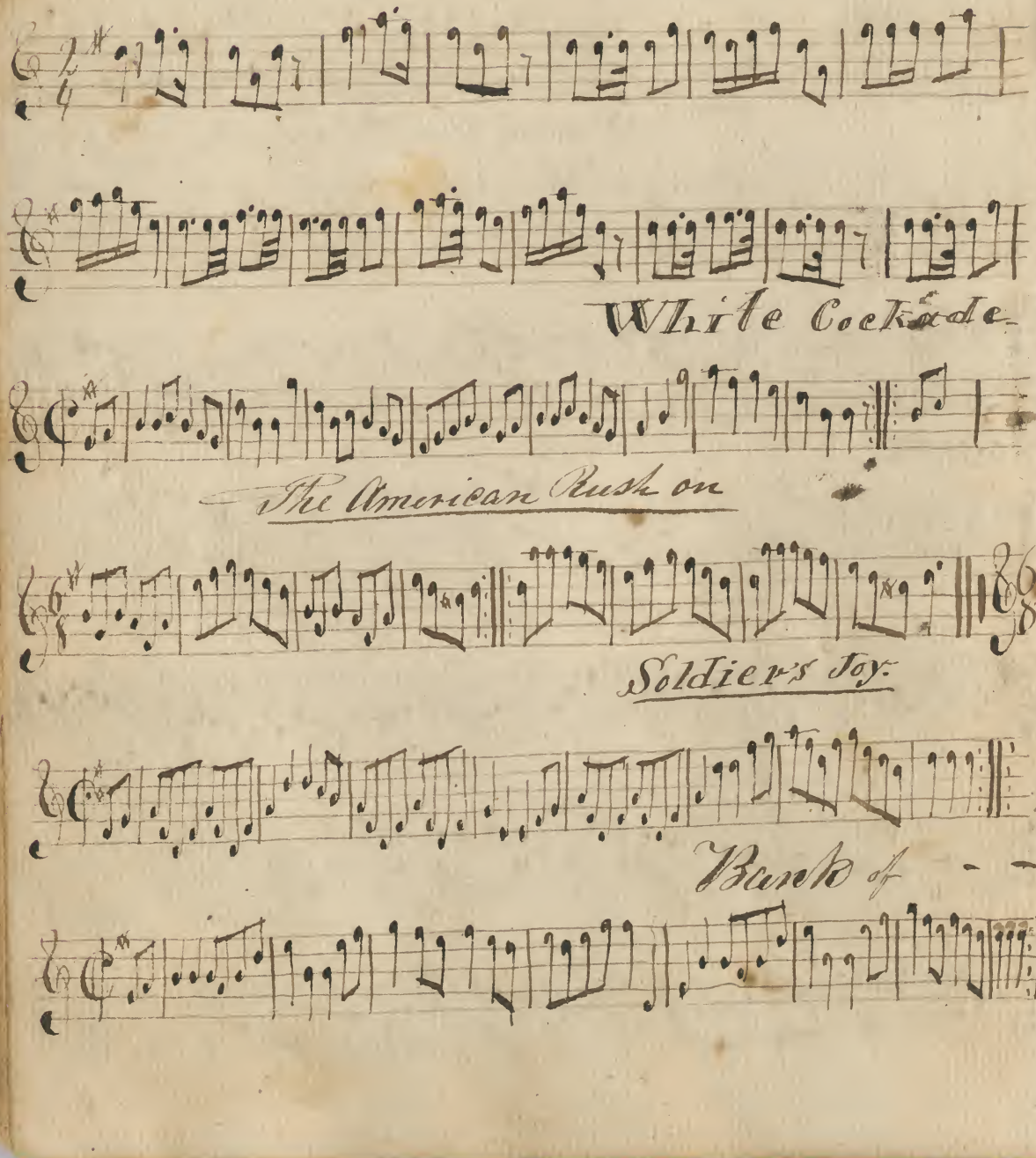

Var. 1.
S.
Var. 4.
Var. 7.
D.C. Var. 1.
Var. 12.
D.C. 8va
Var. 15.
D.C. 8va
Var. 19.
D.C. 8va
Devil's Dream.

Grand St. Spy.

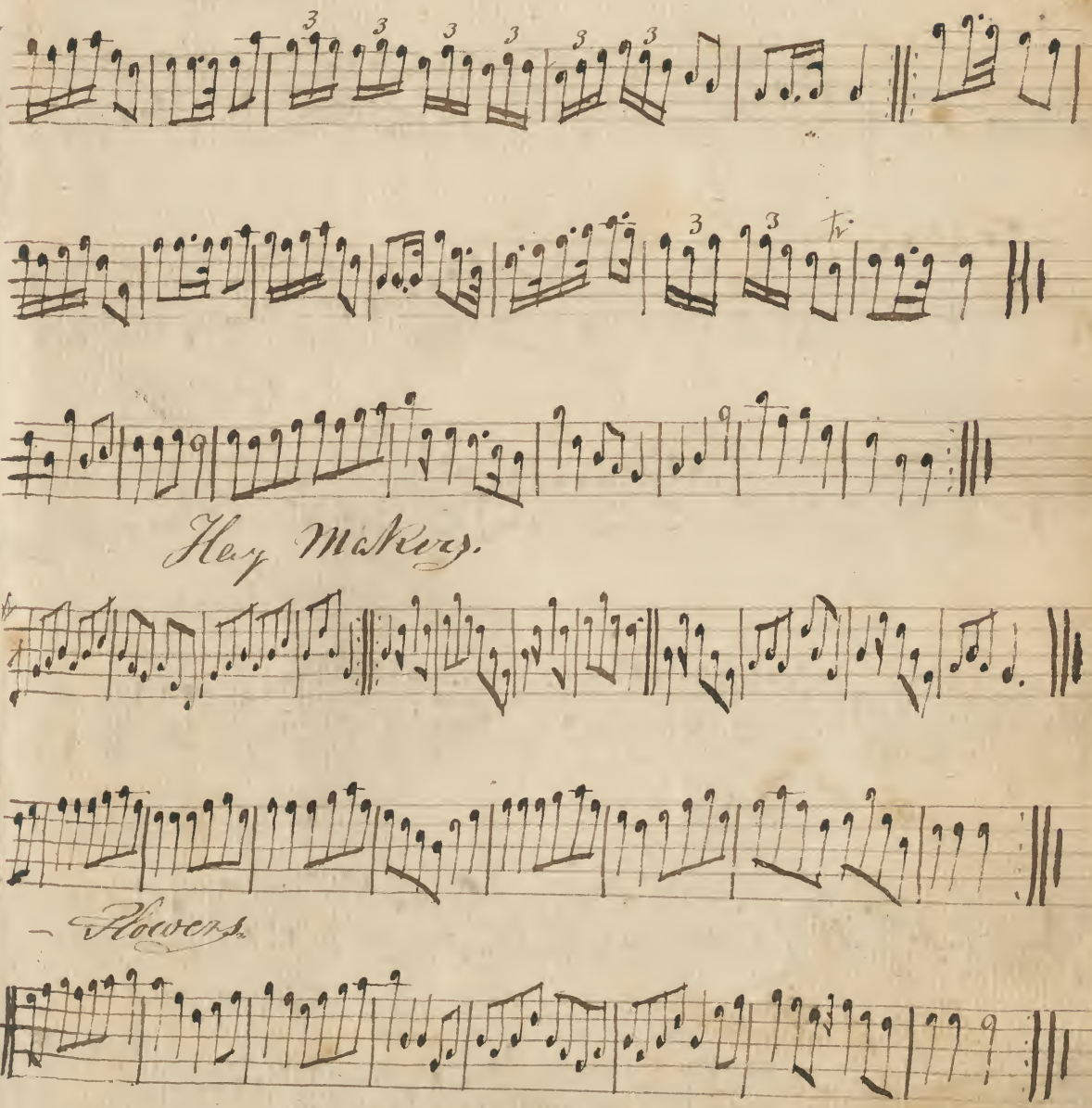



Speed the Plough.
March
Double Do.

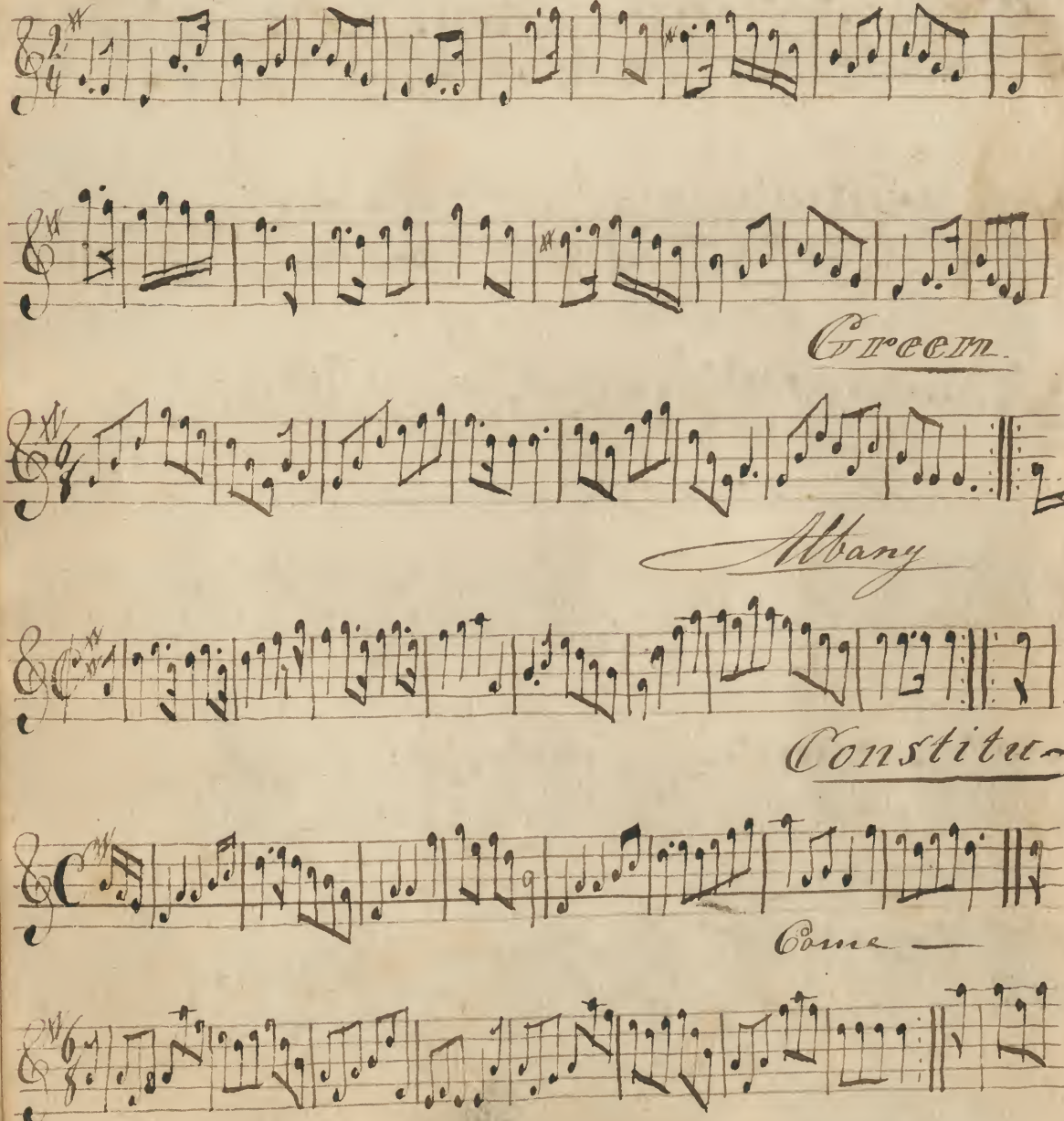
Hail!!!



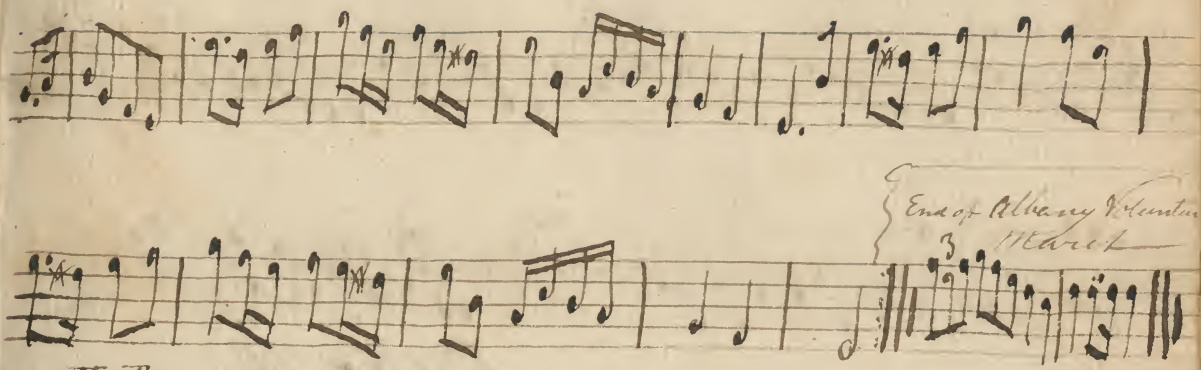
Columbia.


Hay Makers.
Flowers.

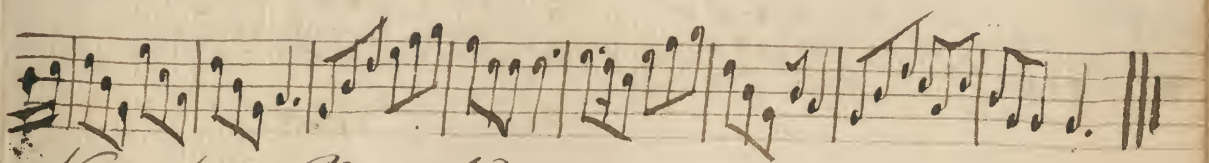
Roslin



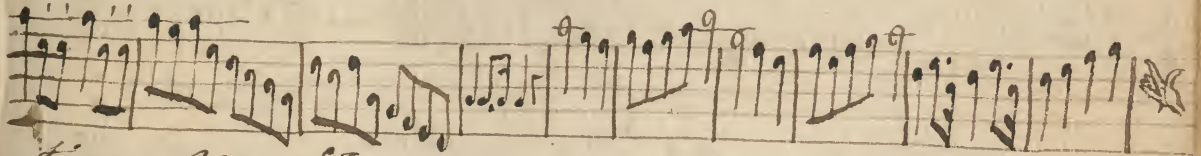
Castle.



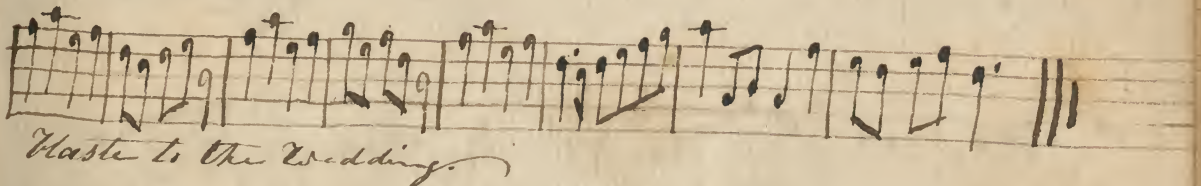
Joke.



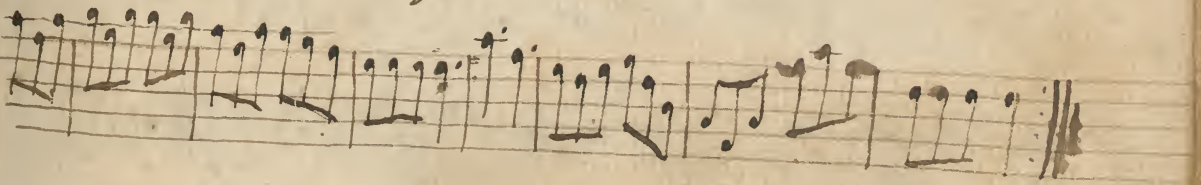
Volunteer March.



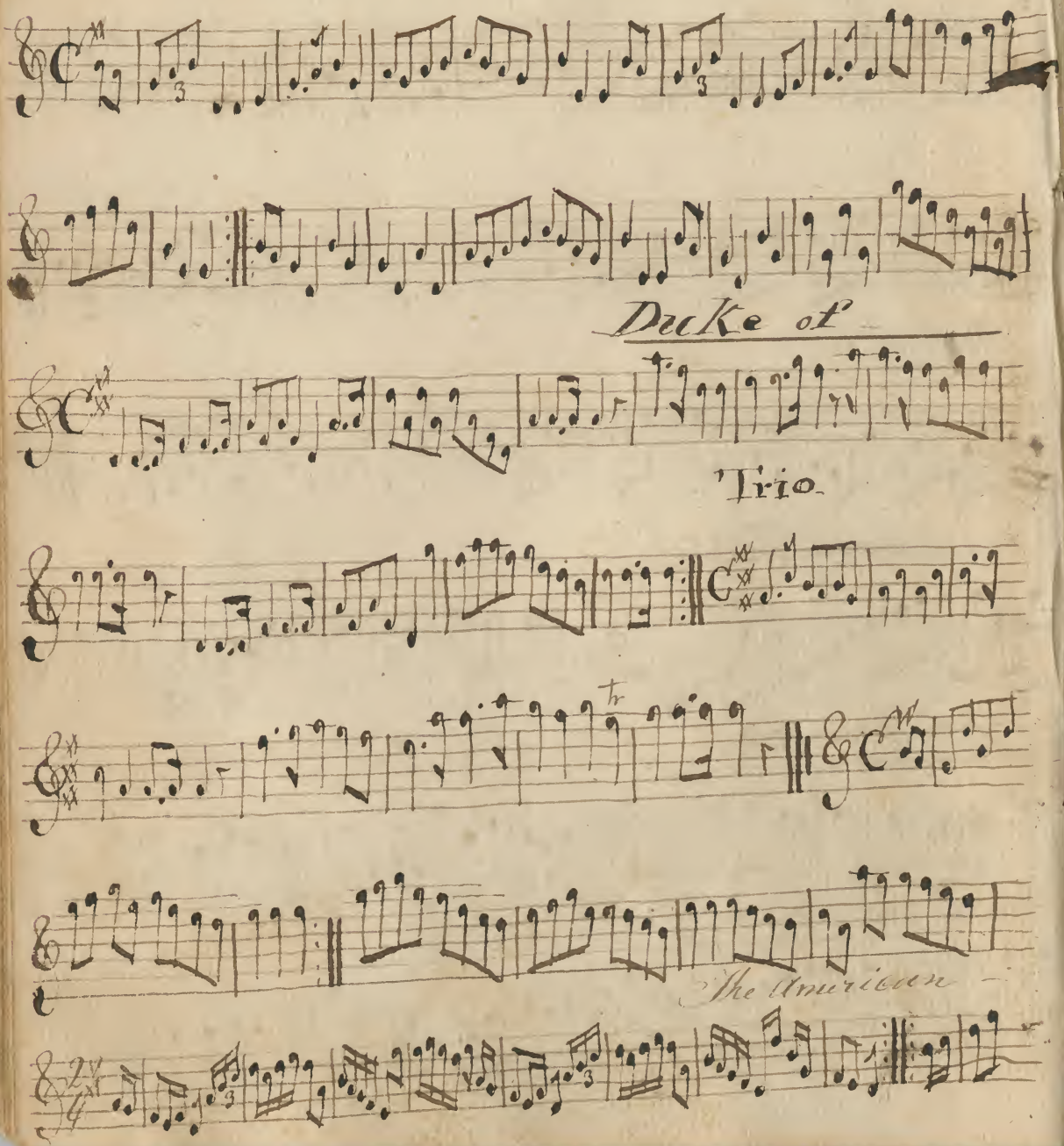
-tion March.



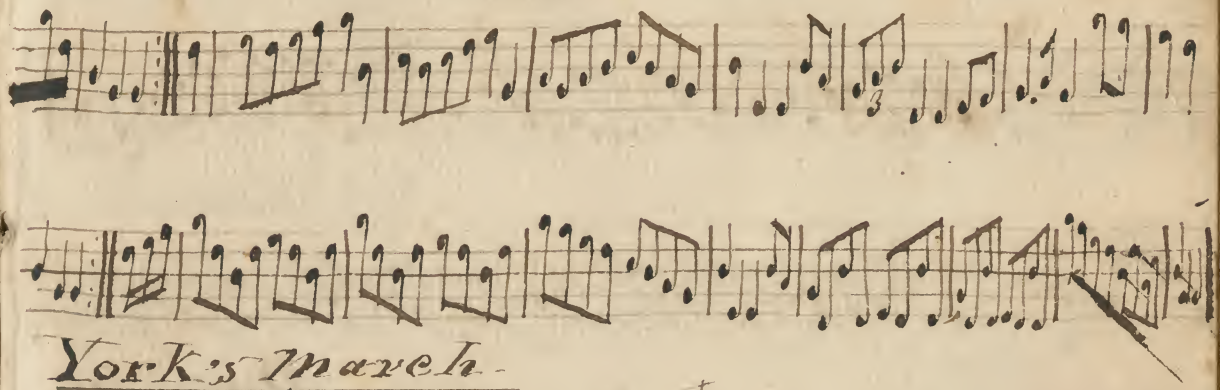
Haste to the Wedding.



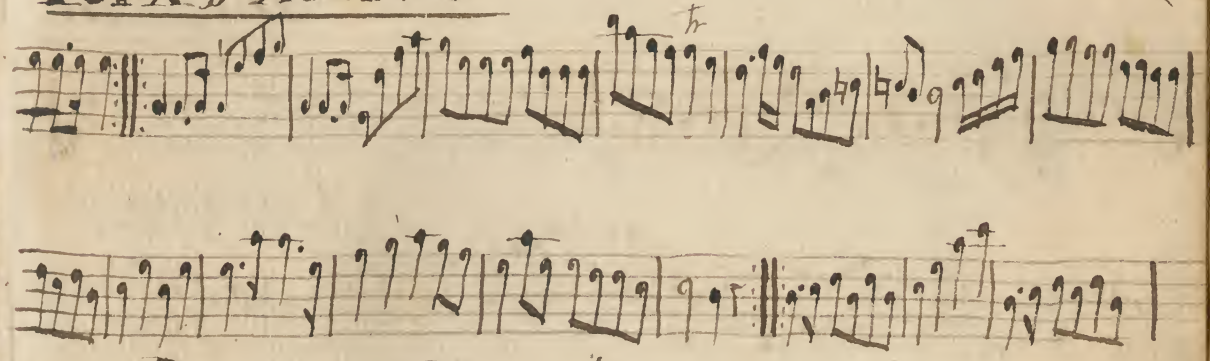
Genl Gouv's



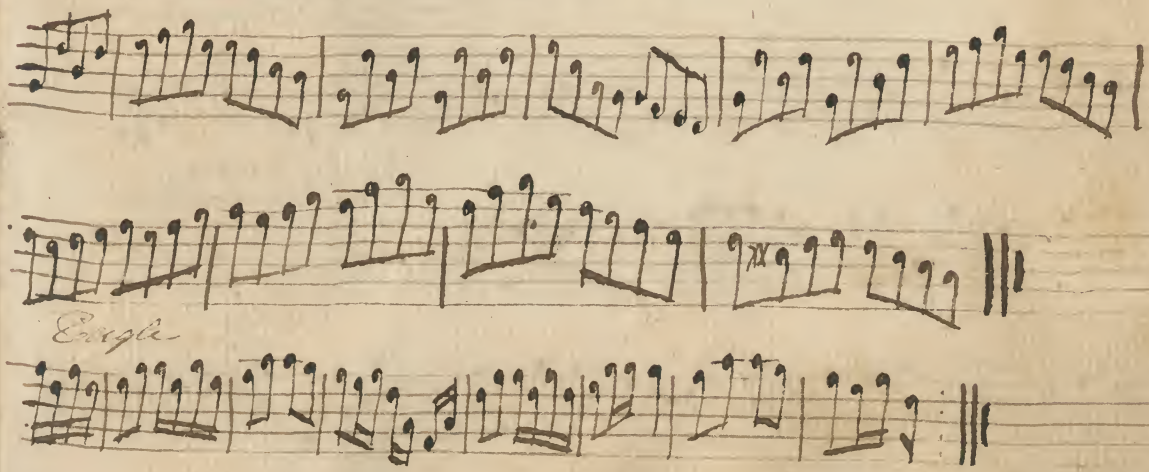
March



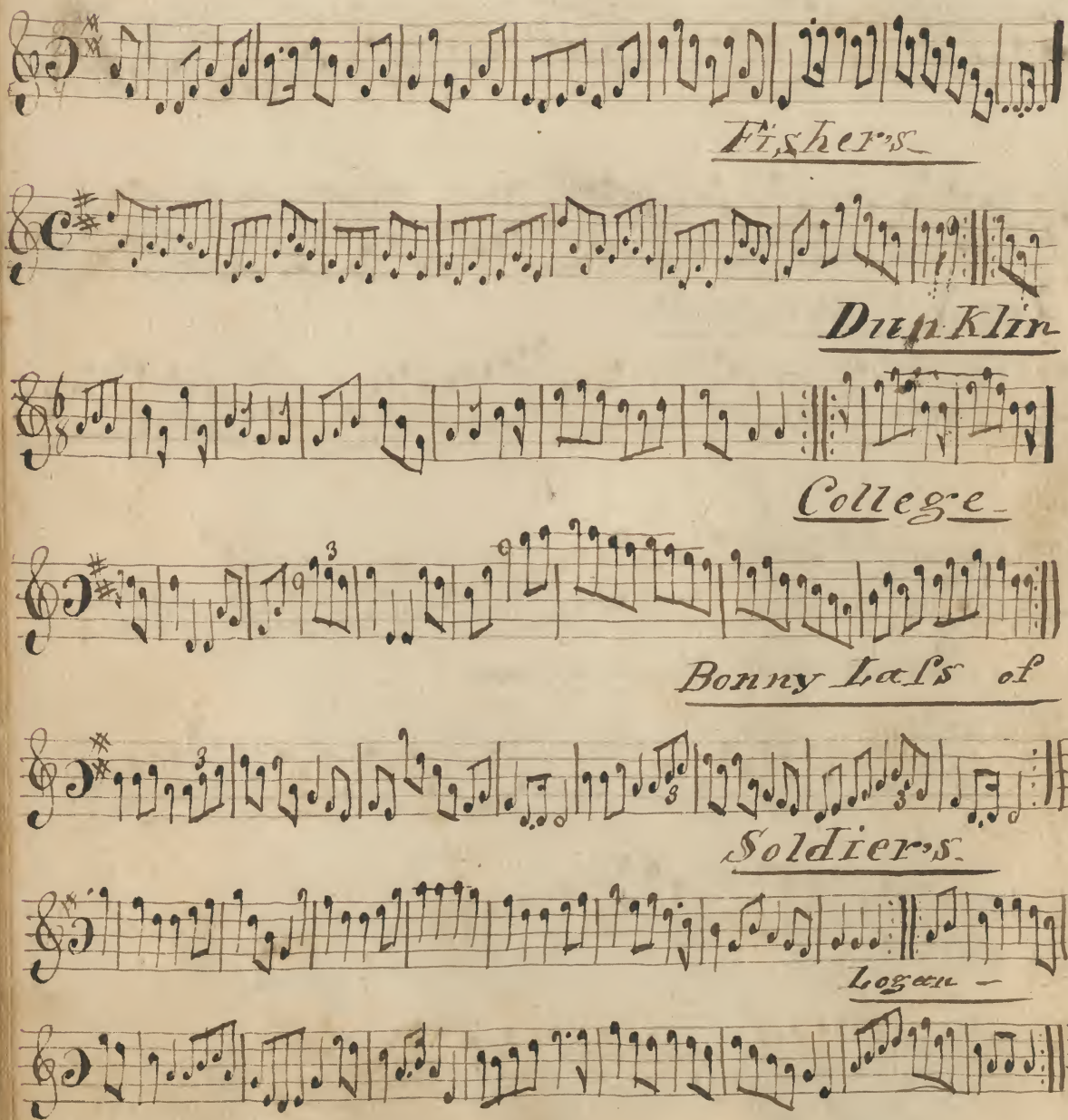
York's March



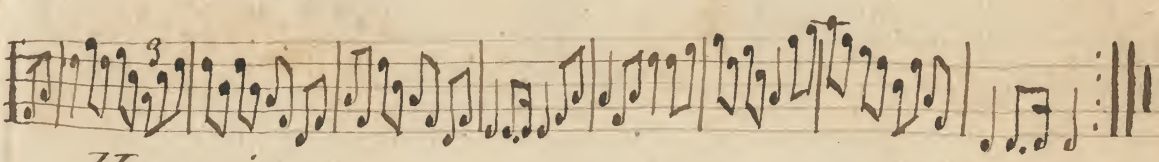
Durang's Hornpip



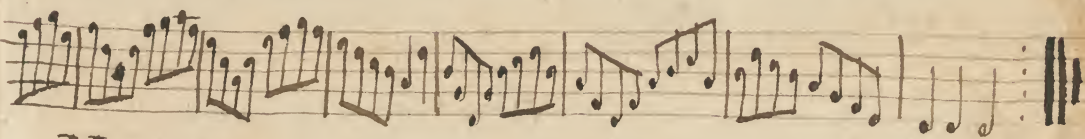
Northampton



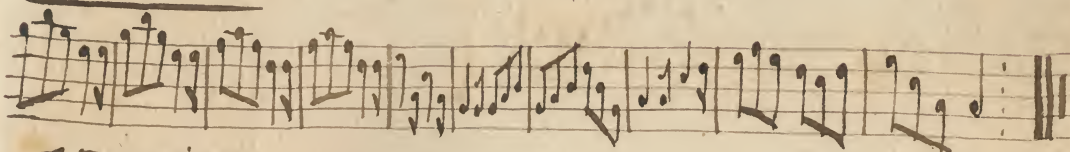
Matrols.



Hornpipe.

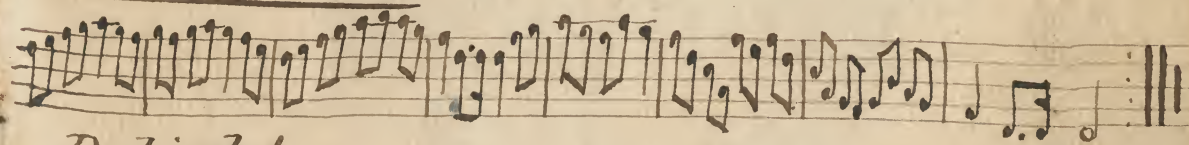


House.

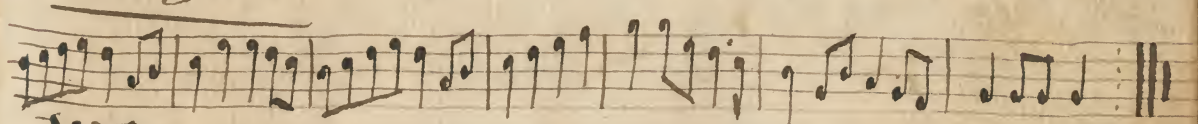


Horpipe.

Aberdeen.



Delight.



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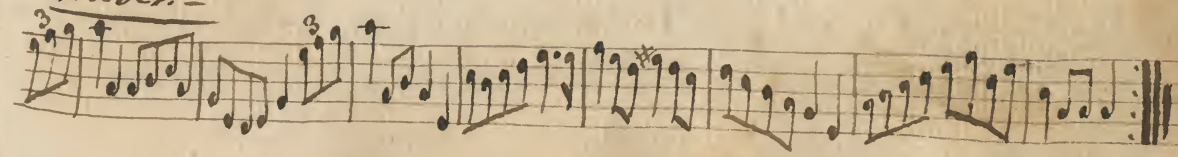

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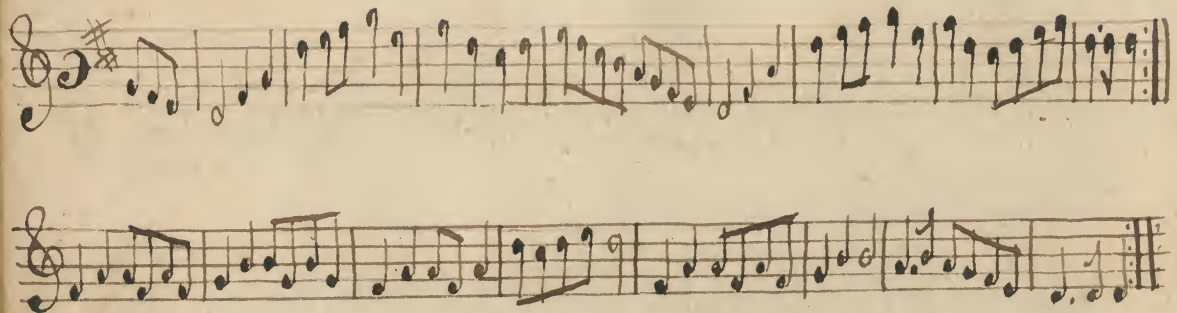

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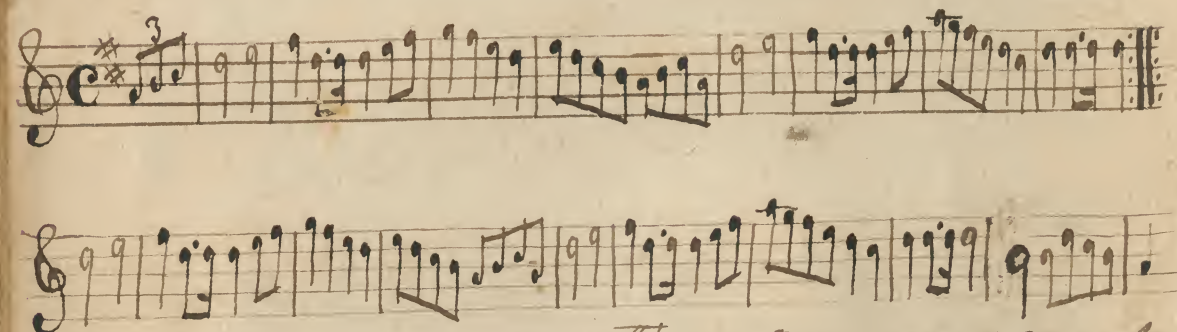
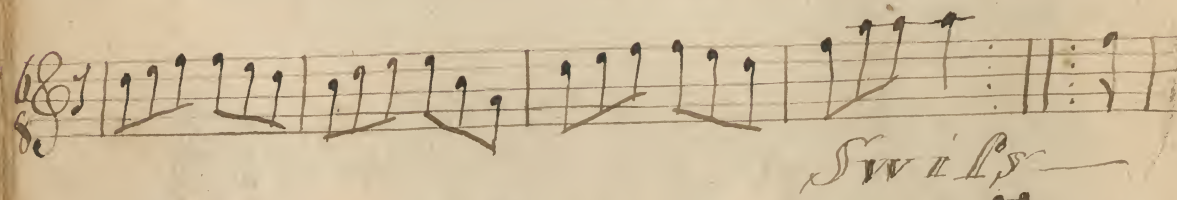


Free Masons March



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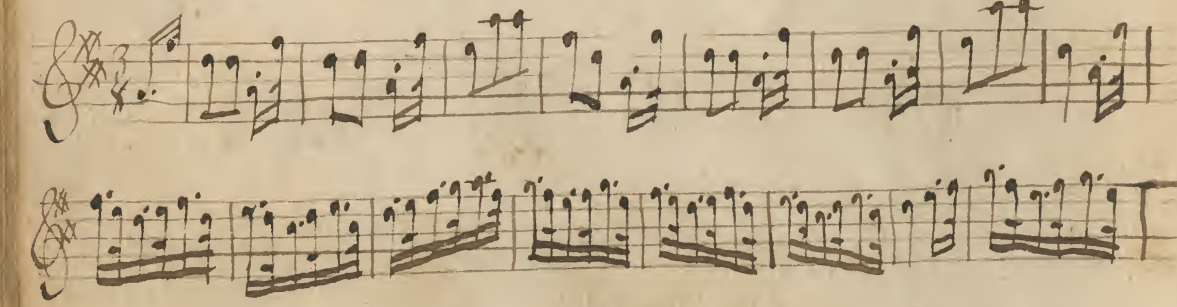

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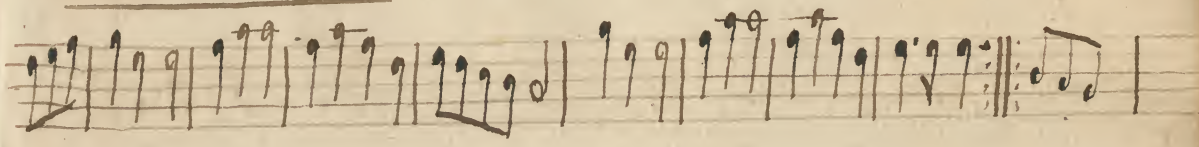

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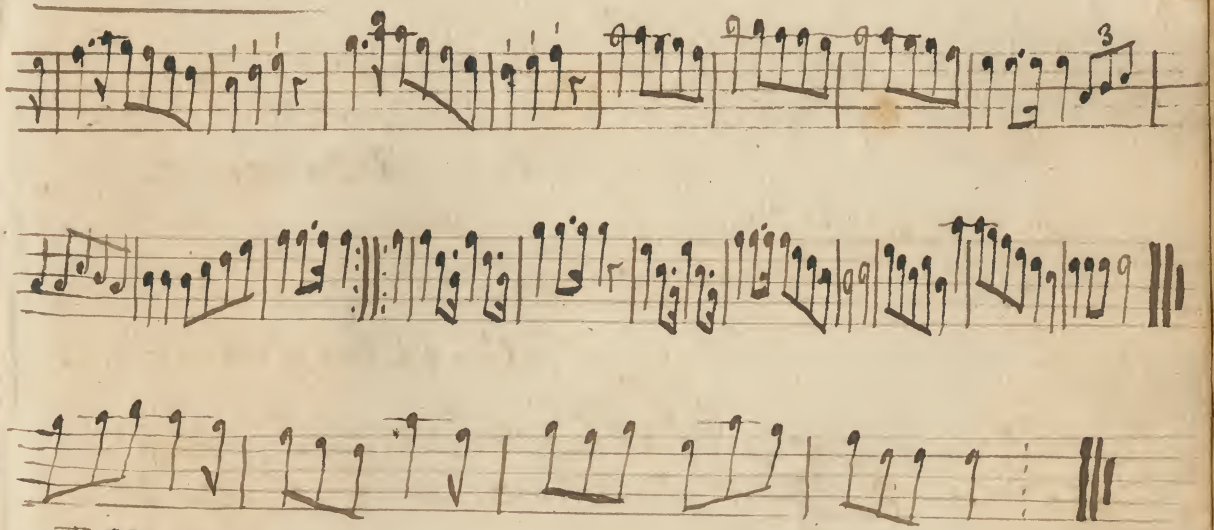
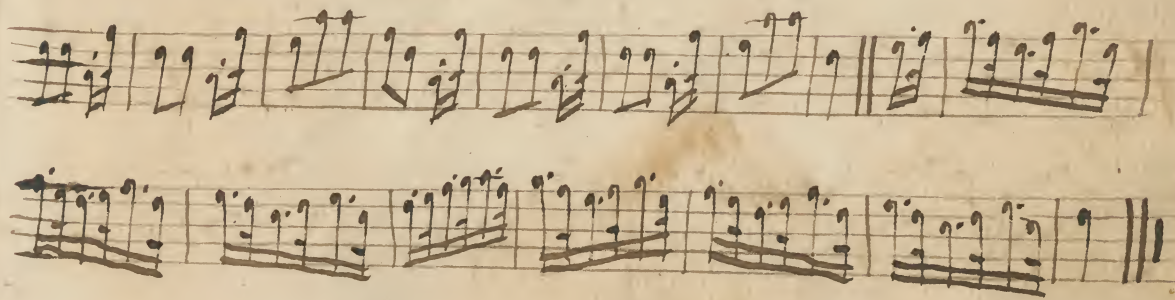
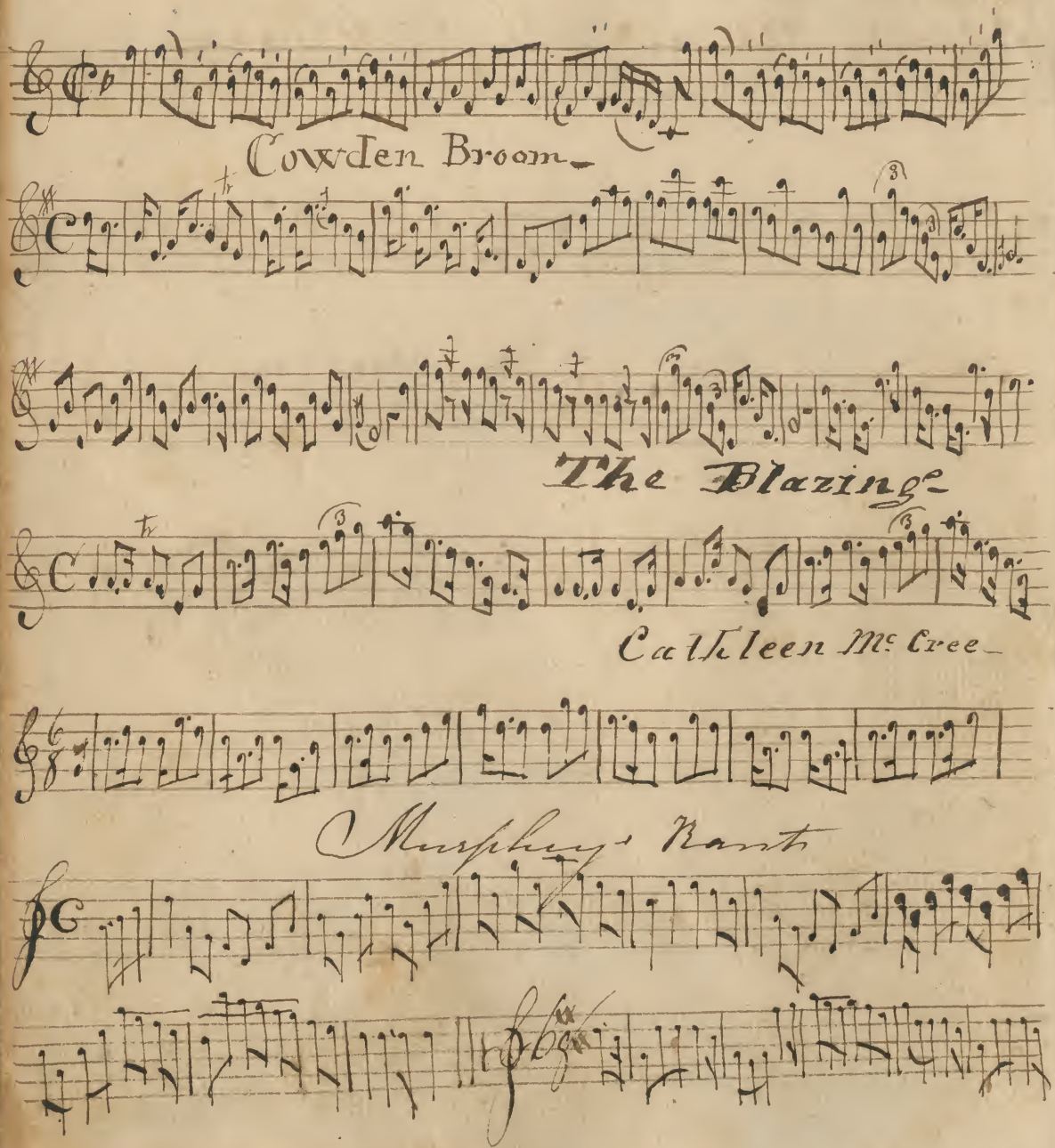


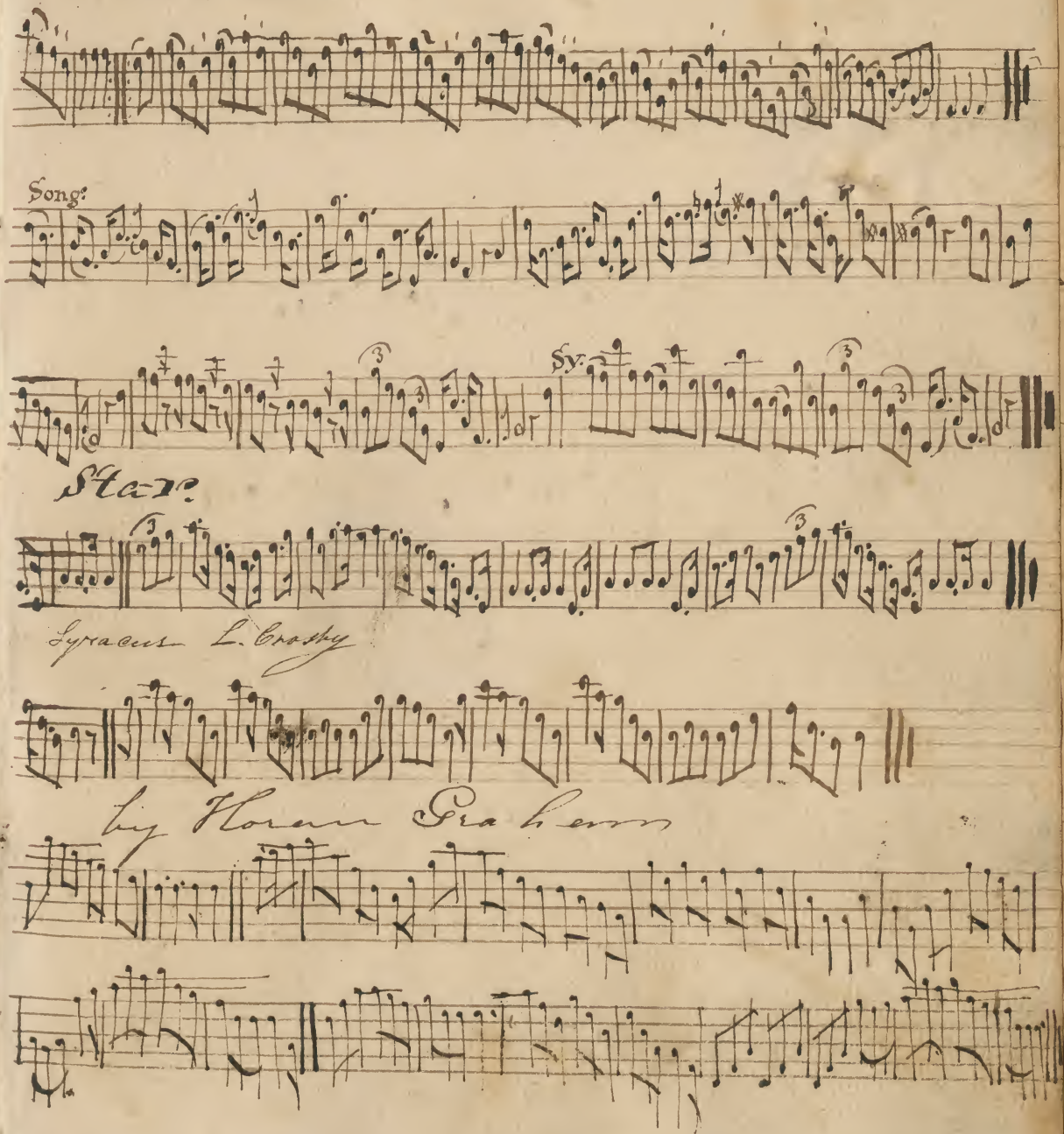
Waltz.



Fisher's

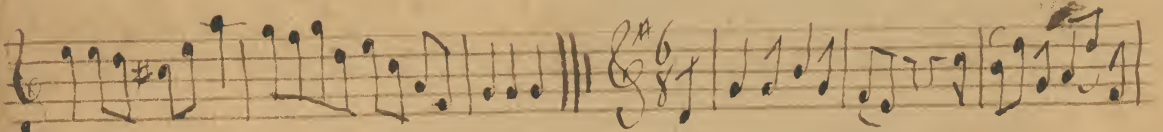


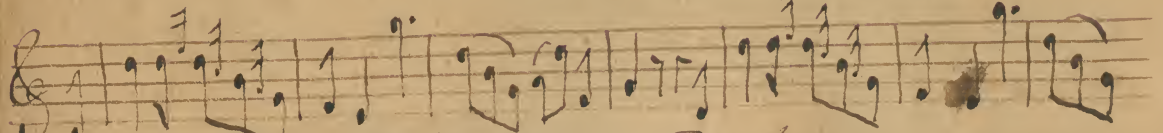
Hornpipe



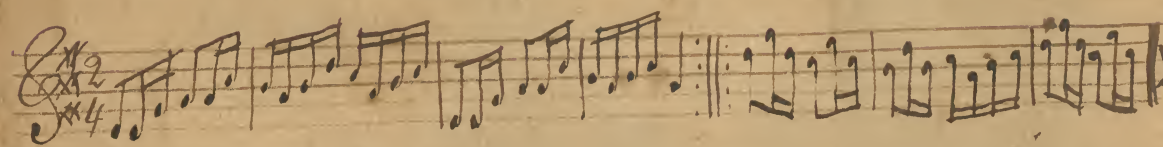
Portsmouth

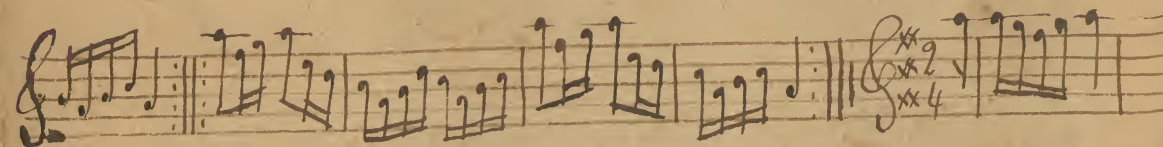
Madrid

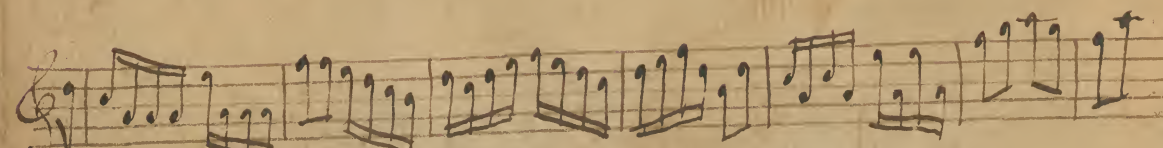




Miss Moore's Rant







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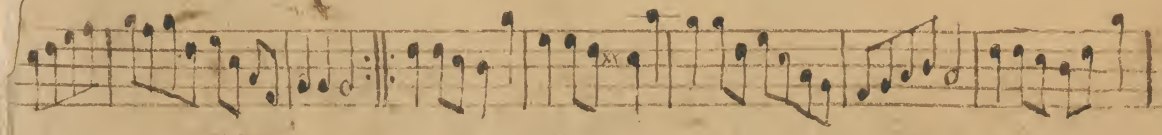

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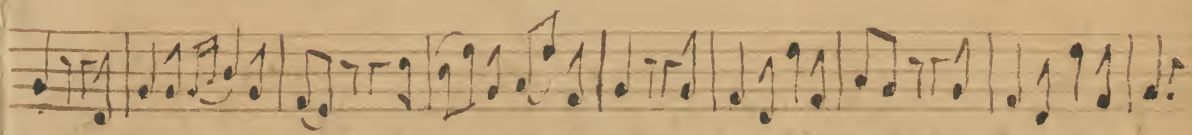
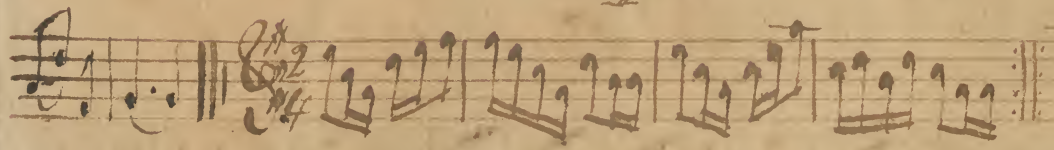
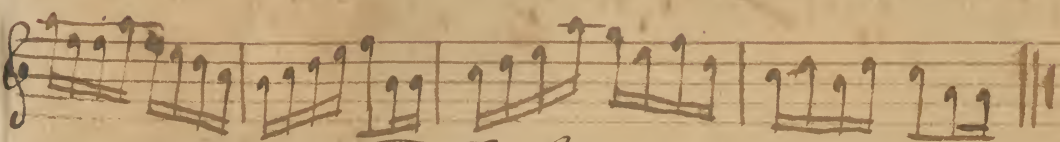
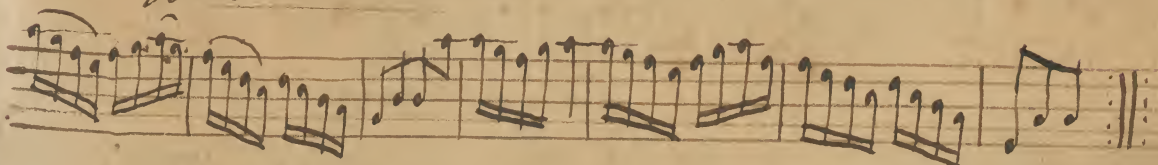


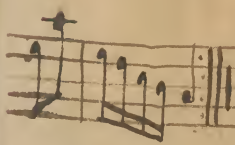
Trip to Holland



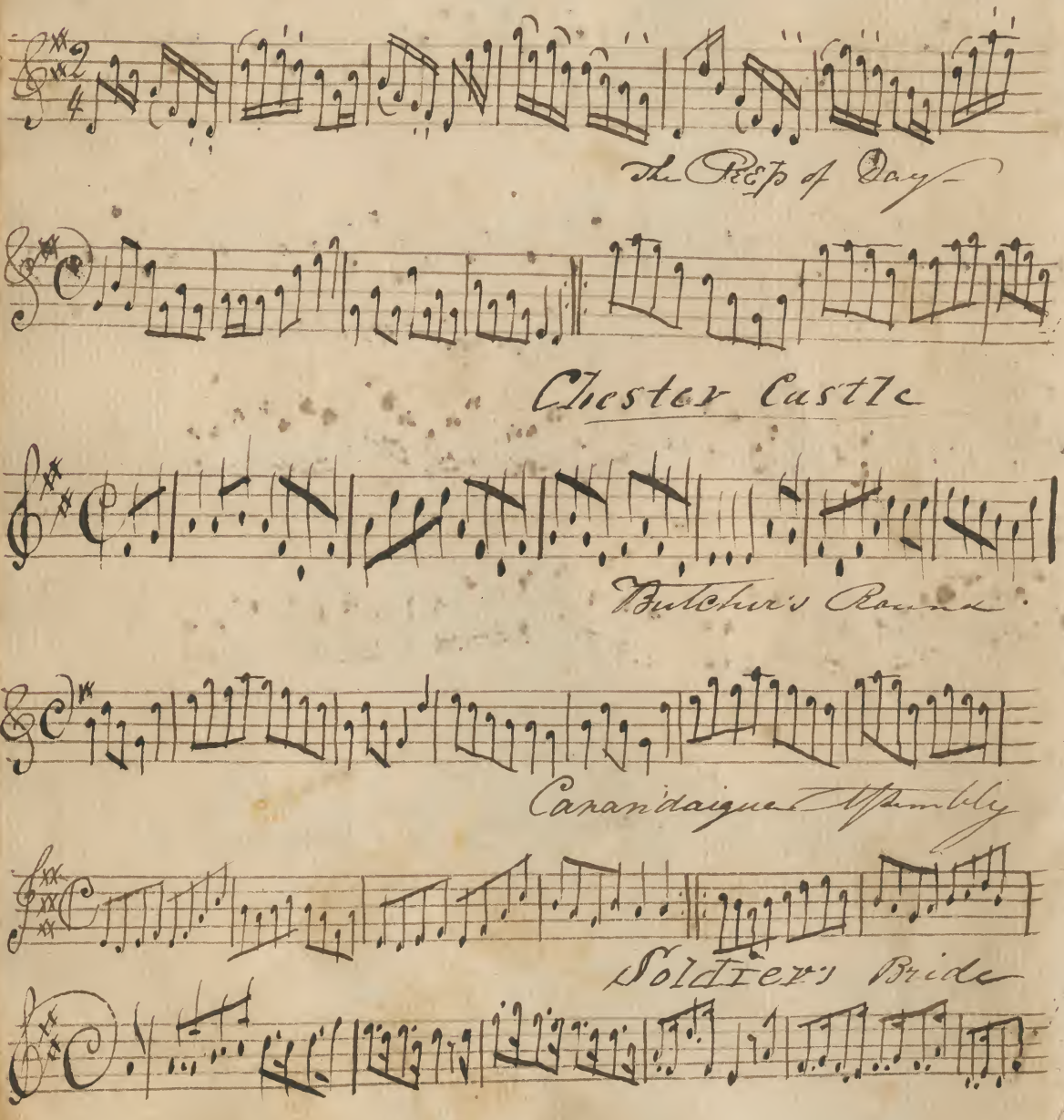


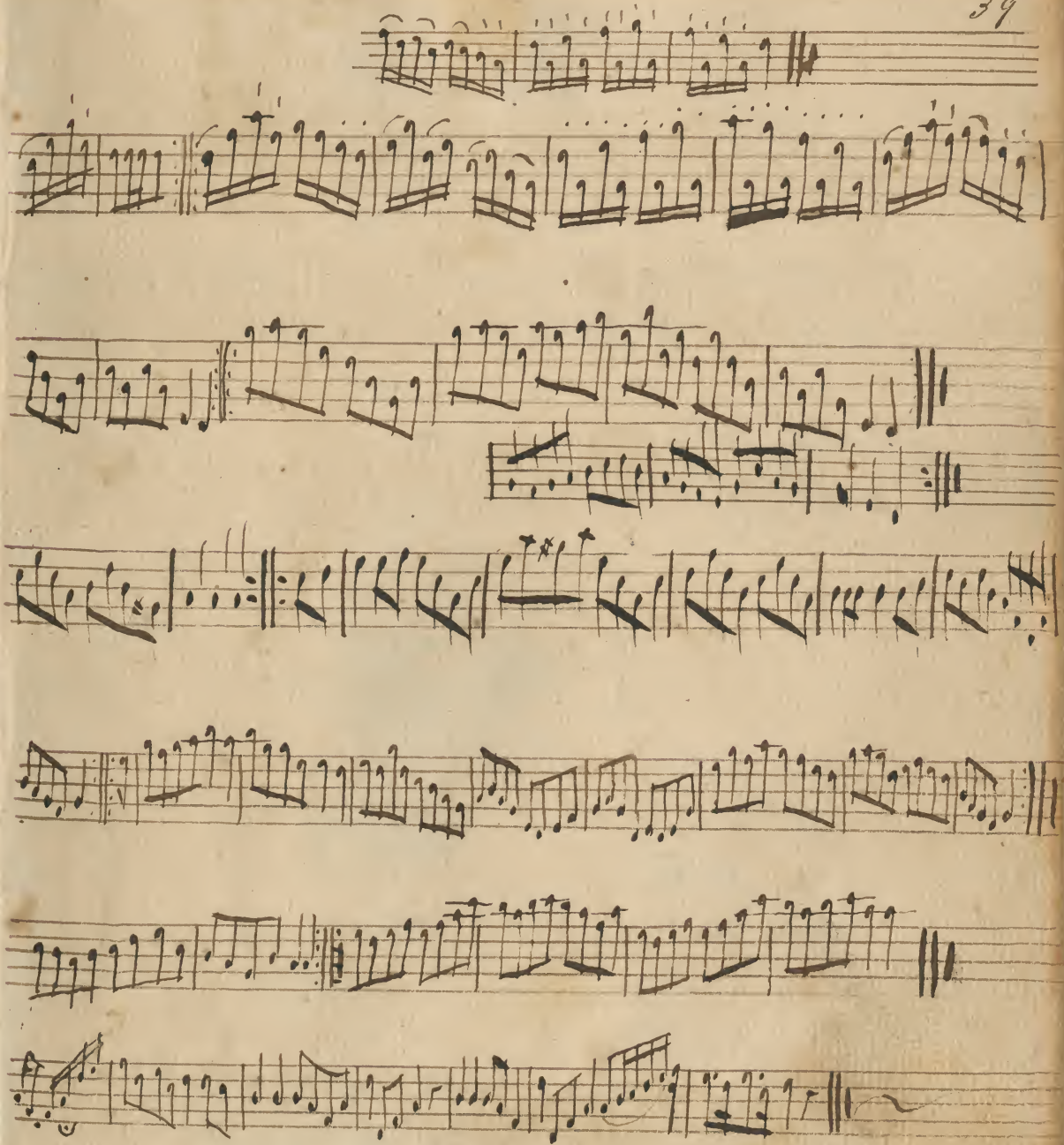
Well done Jack



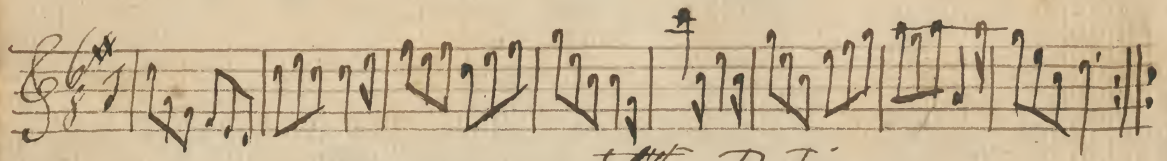


The 12th of March by Whitlock

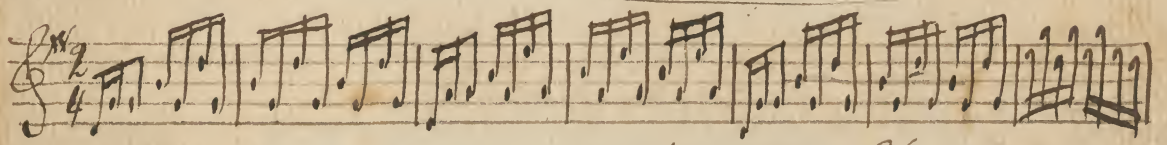




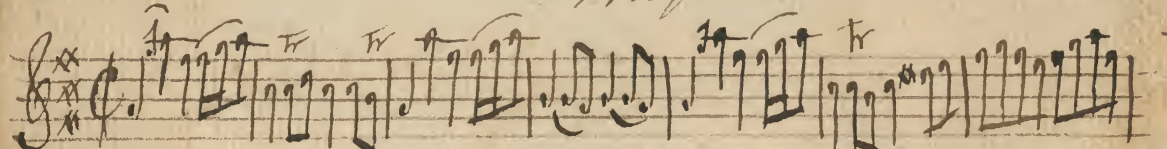
Take care of your cash



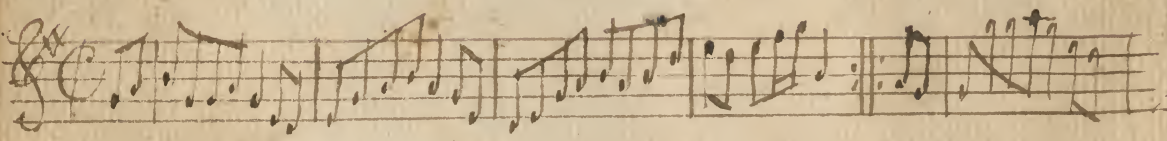
The Robin



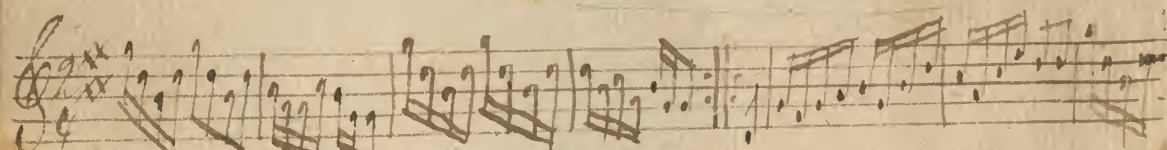
Miss McCloudy Reel



Long Hills of Mourne.



Ricketts Reel.



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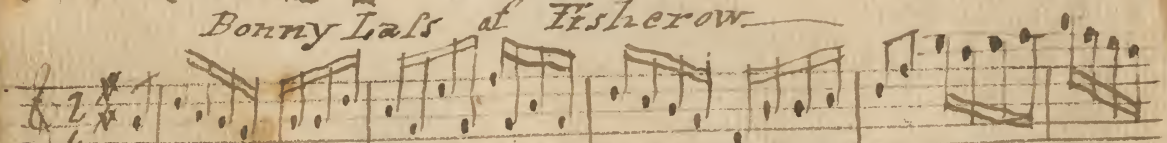

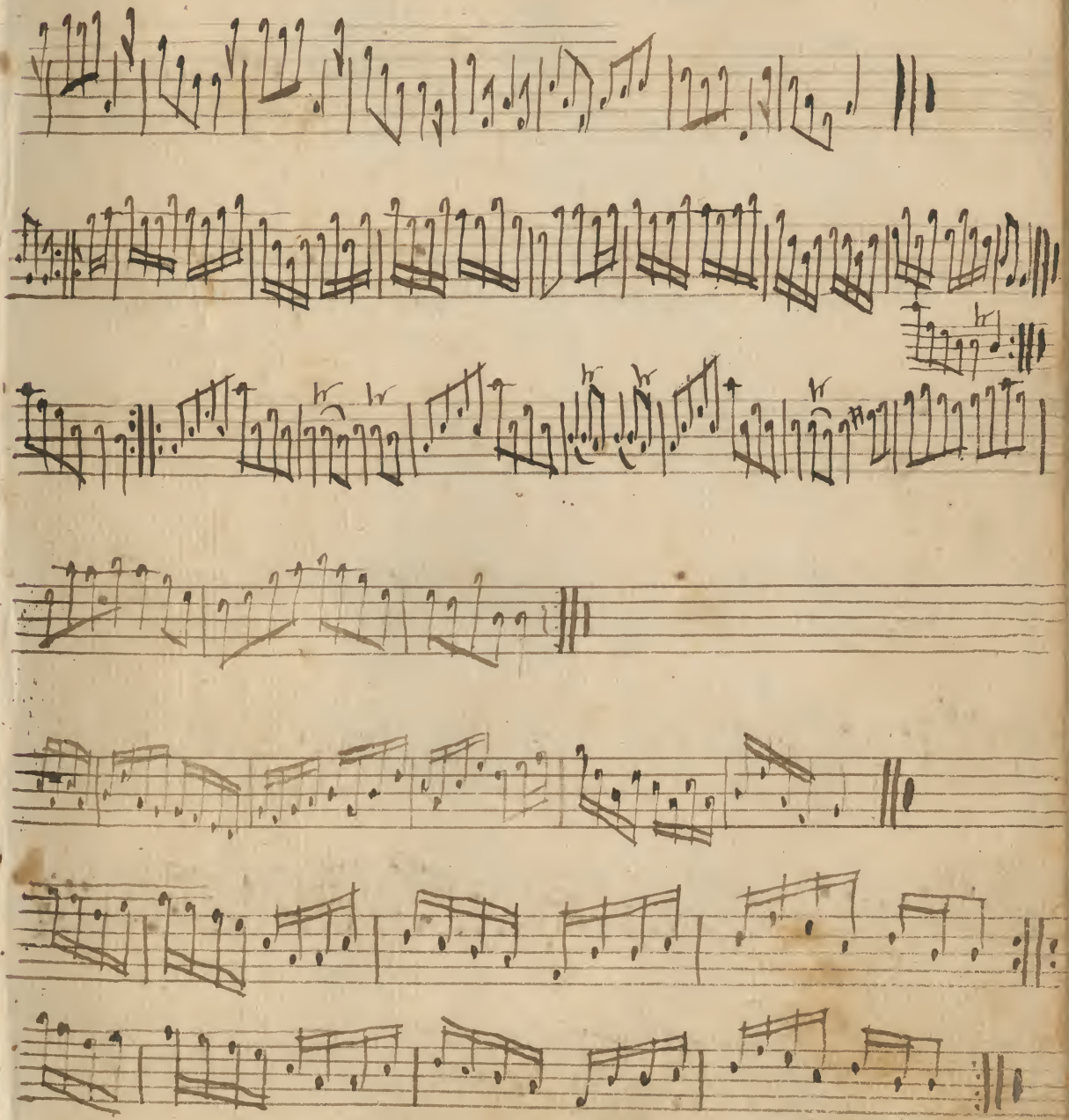

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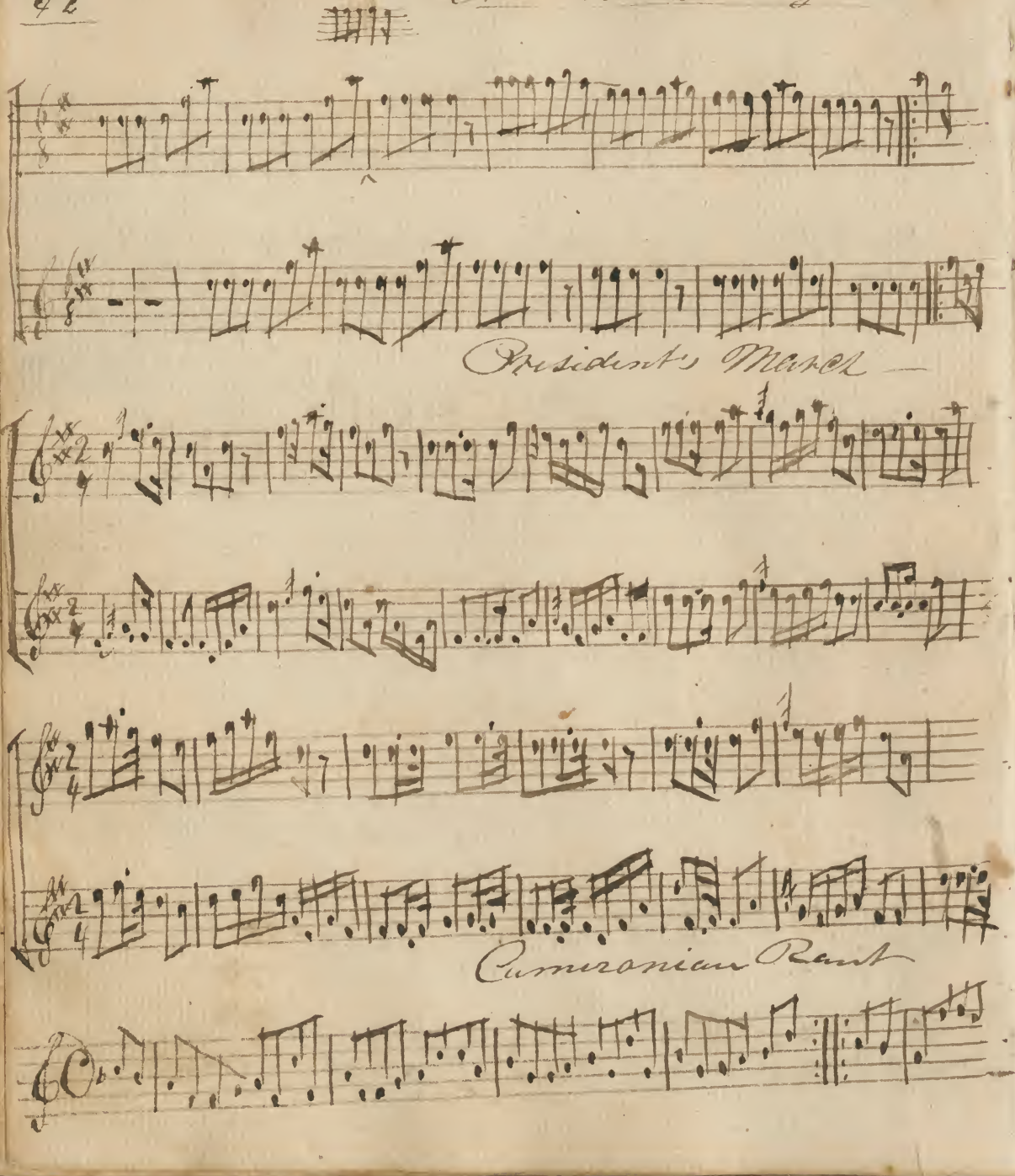

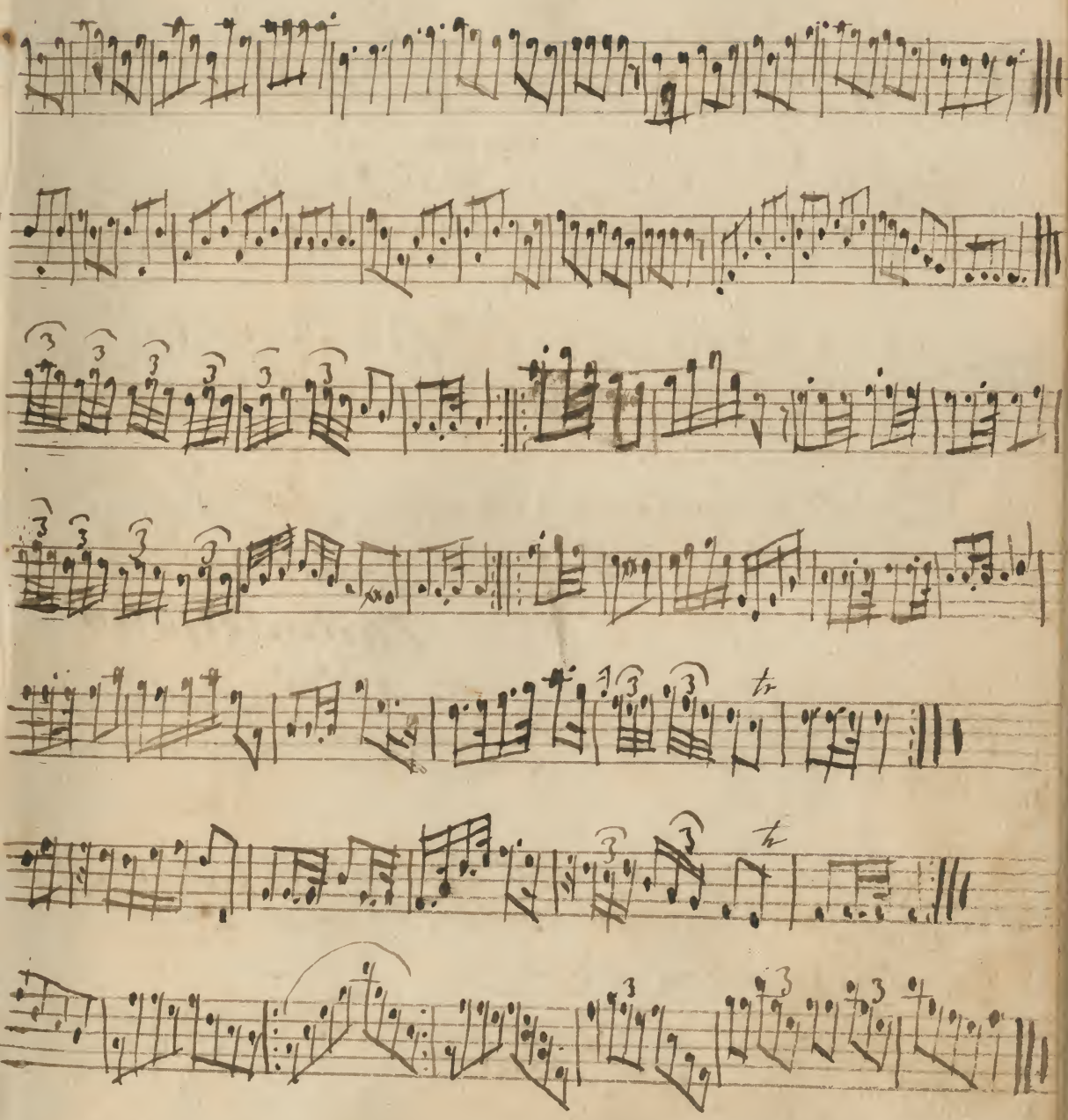
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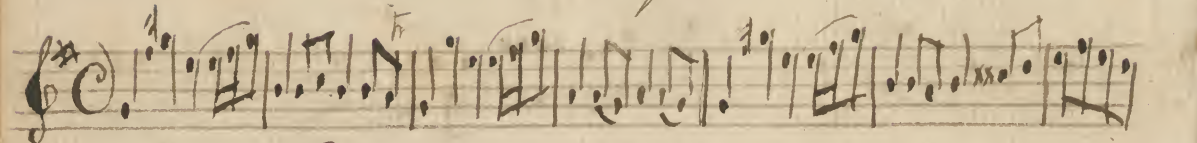
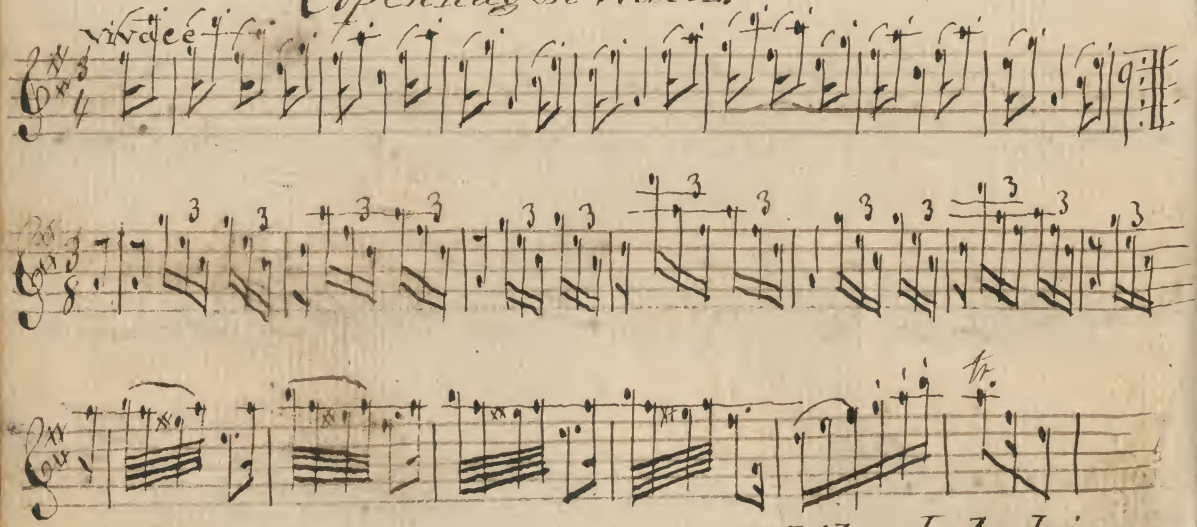
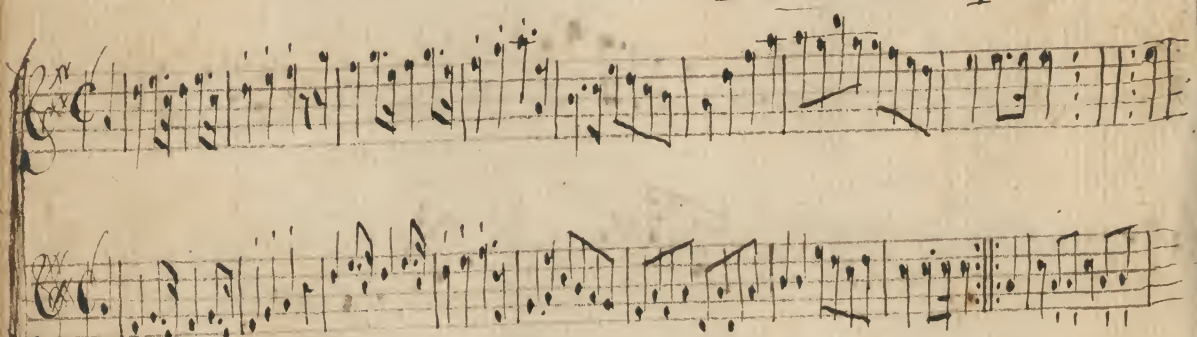
Miss Mc Cloud's



Copenhagen Waltz.



Philadelphia



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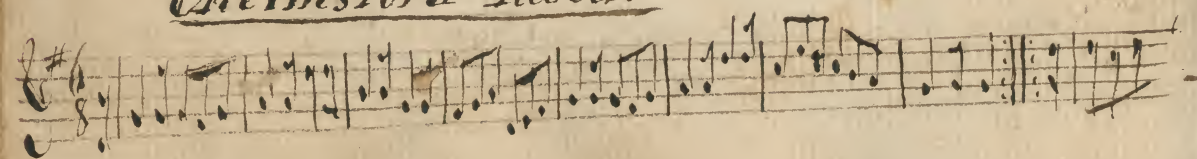

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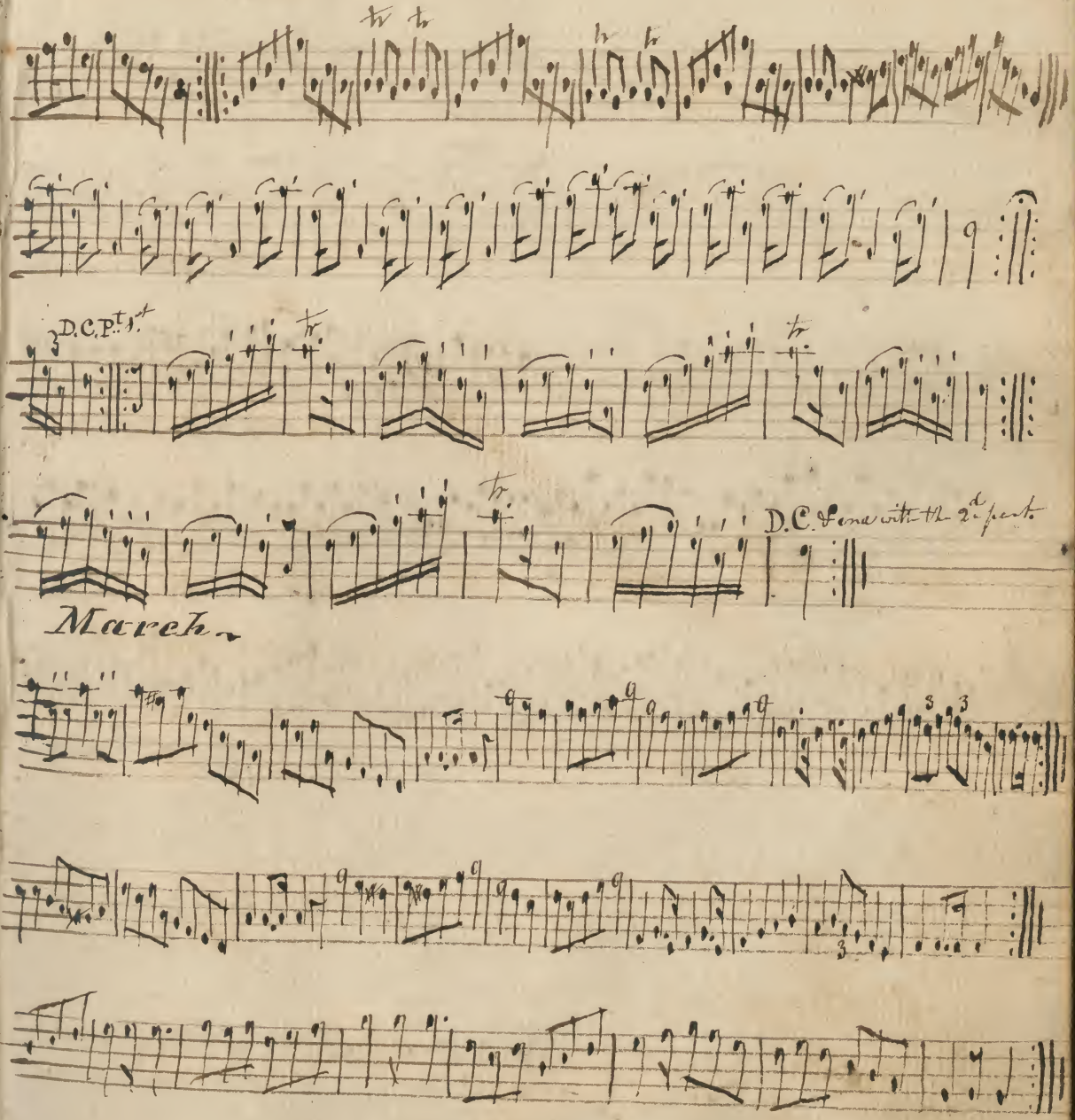

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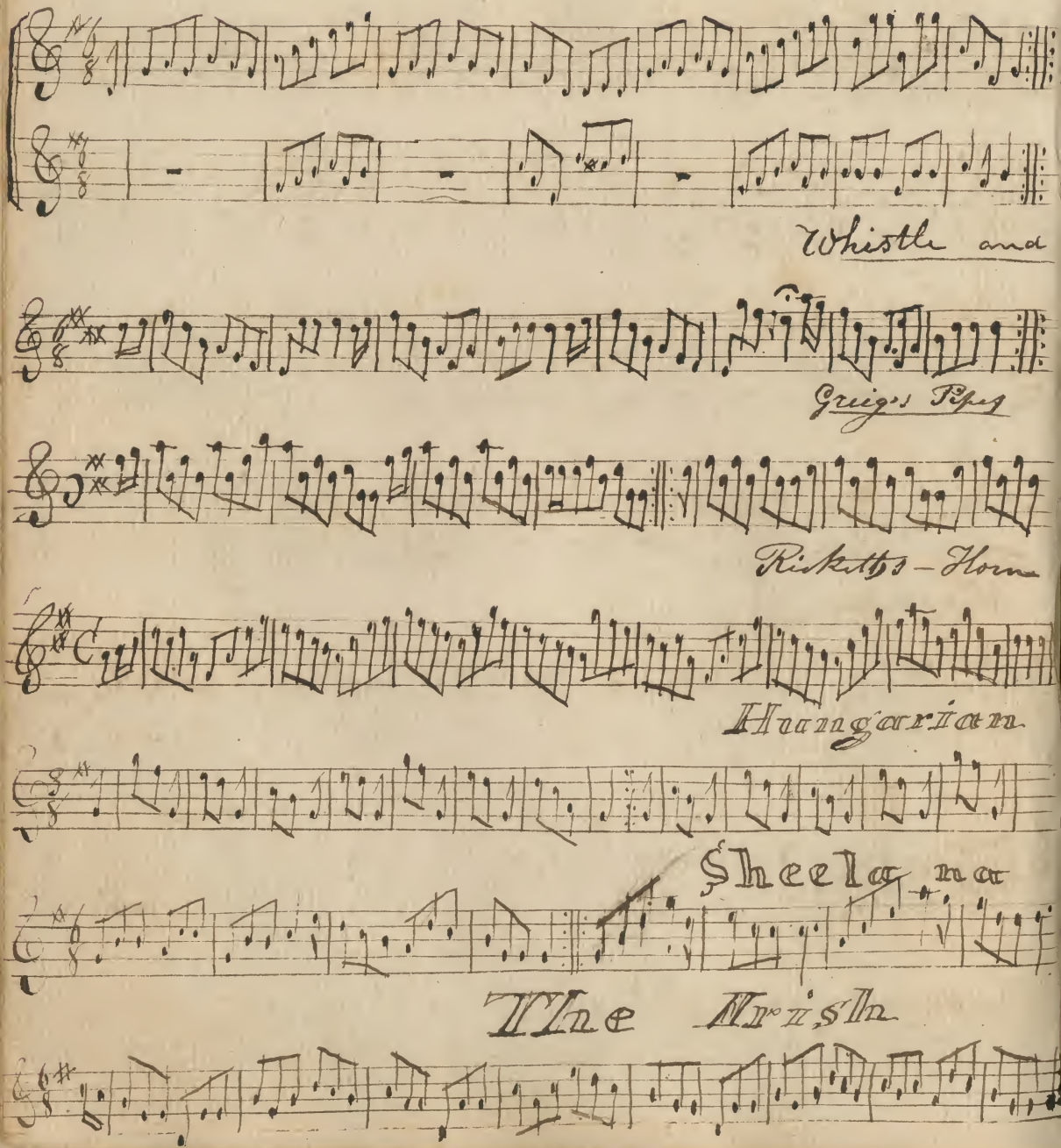
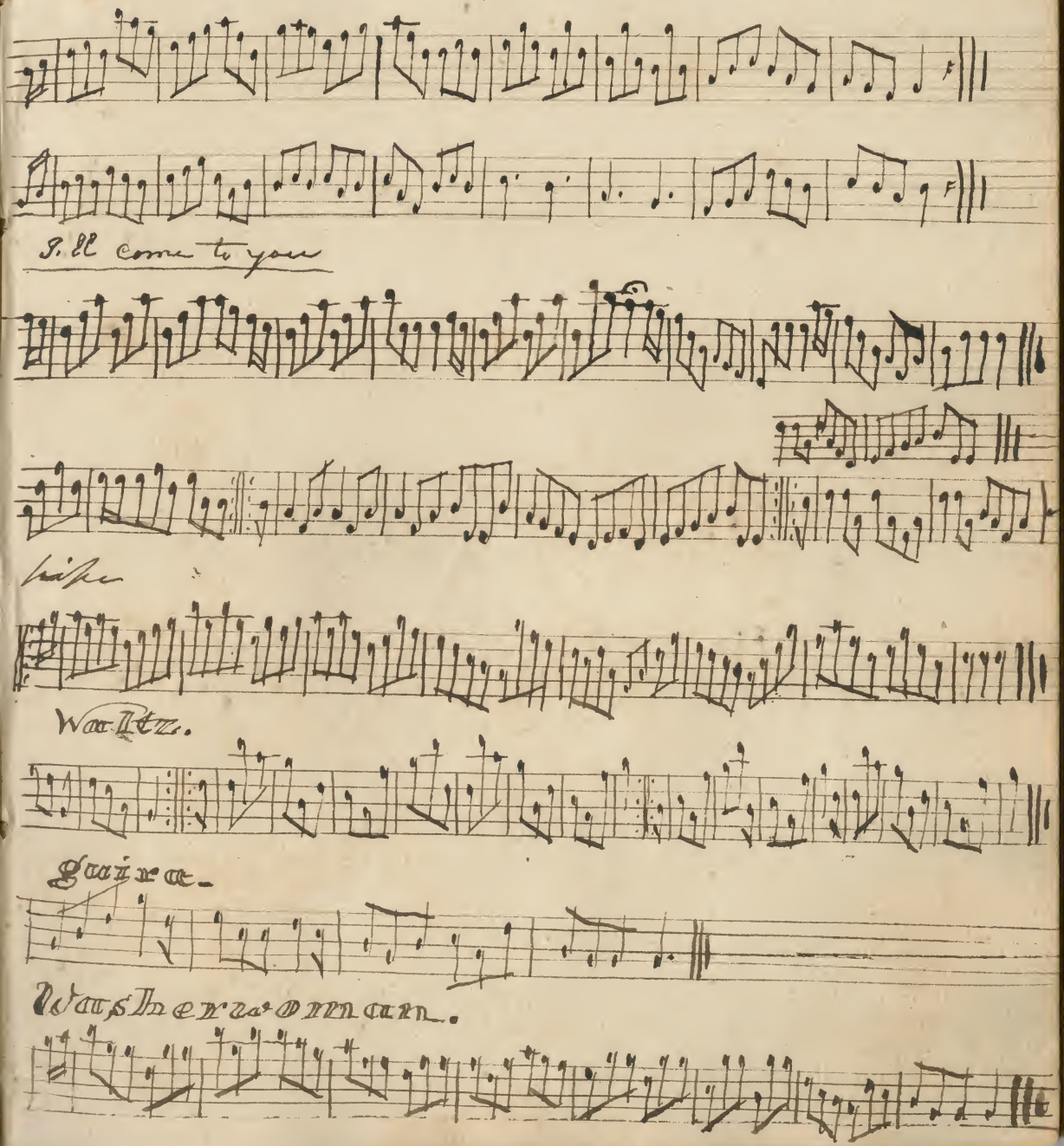

Whistle and
Grieg's Pipes
Ricketts - Horn
Hungarian
Sheela ma
The Irish

Barry's Whim



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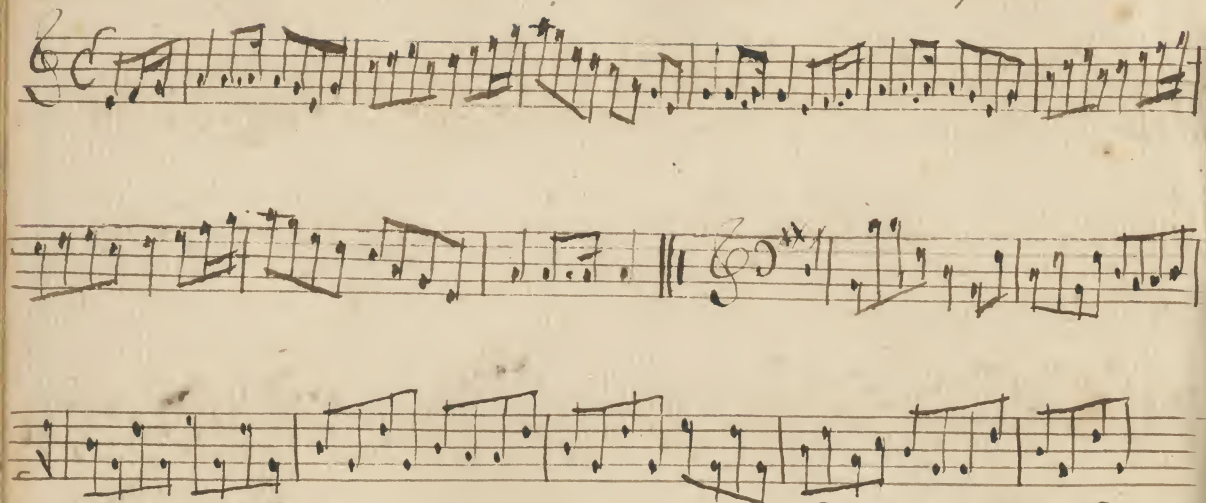
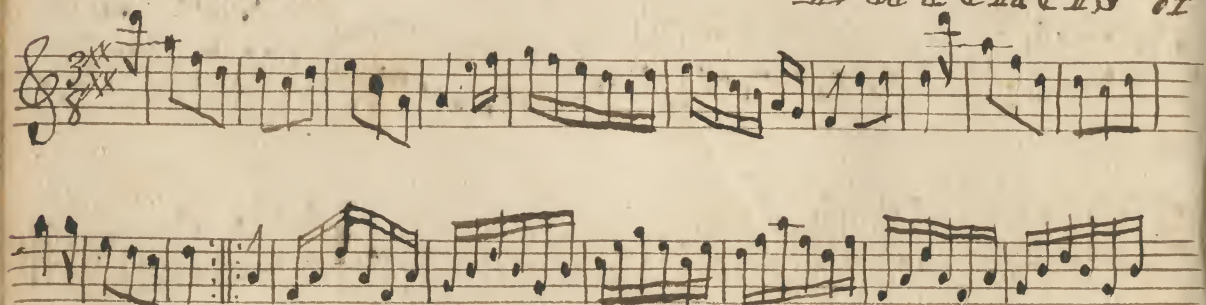
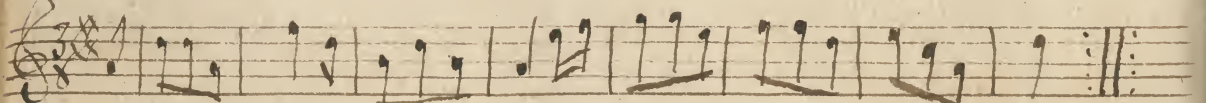


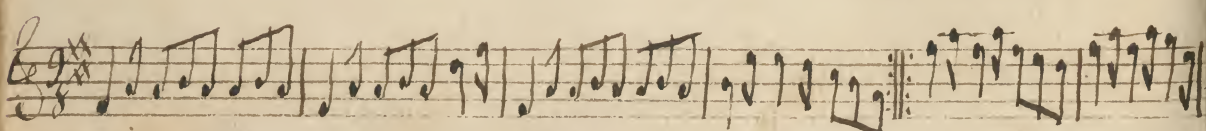
Dutchess of



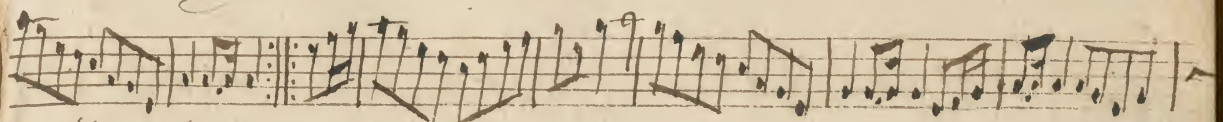
The Cottage



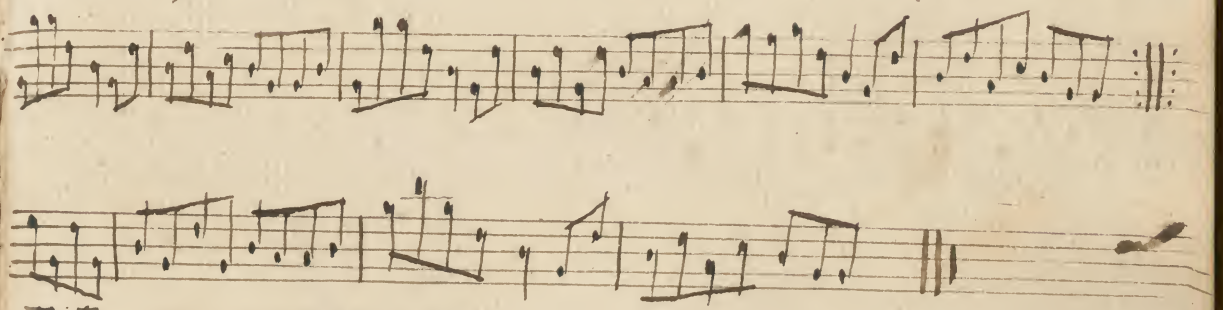
Hewett's



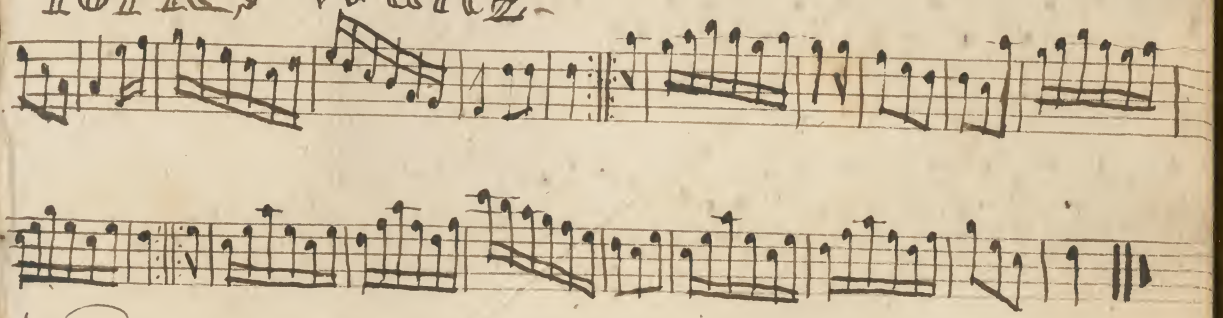
Crossing the Rhine



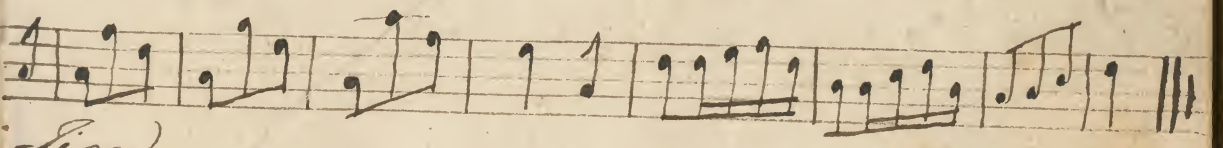
The Lass in your Town



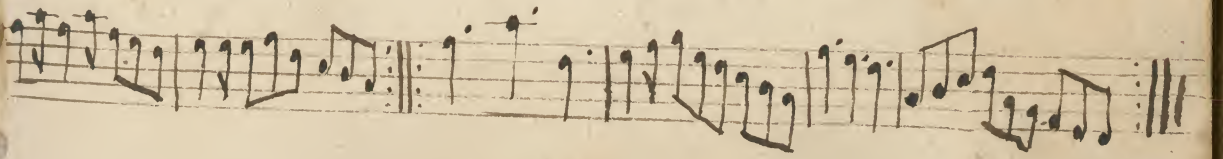
York's Waltz.



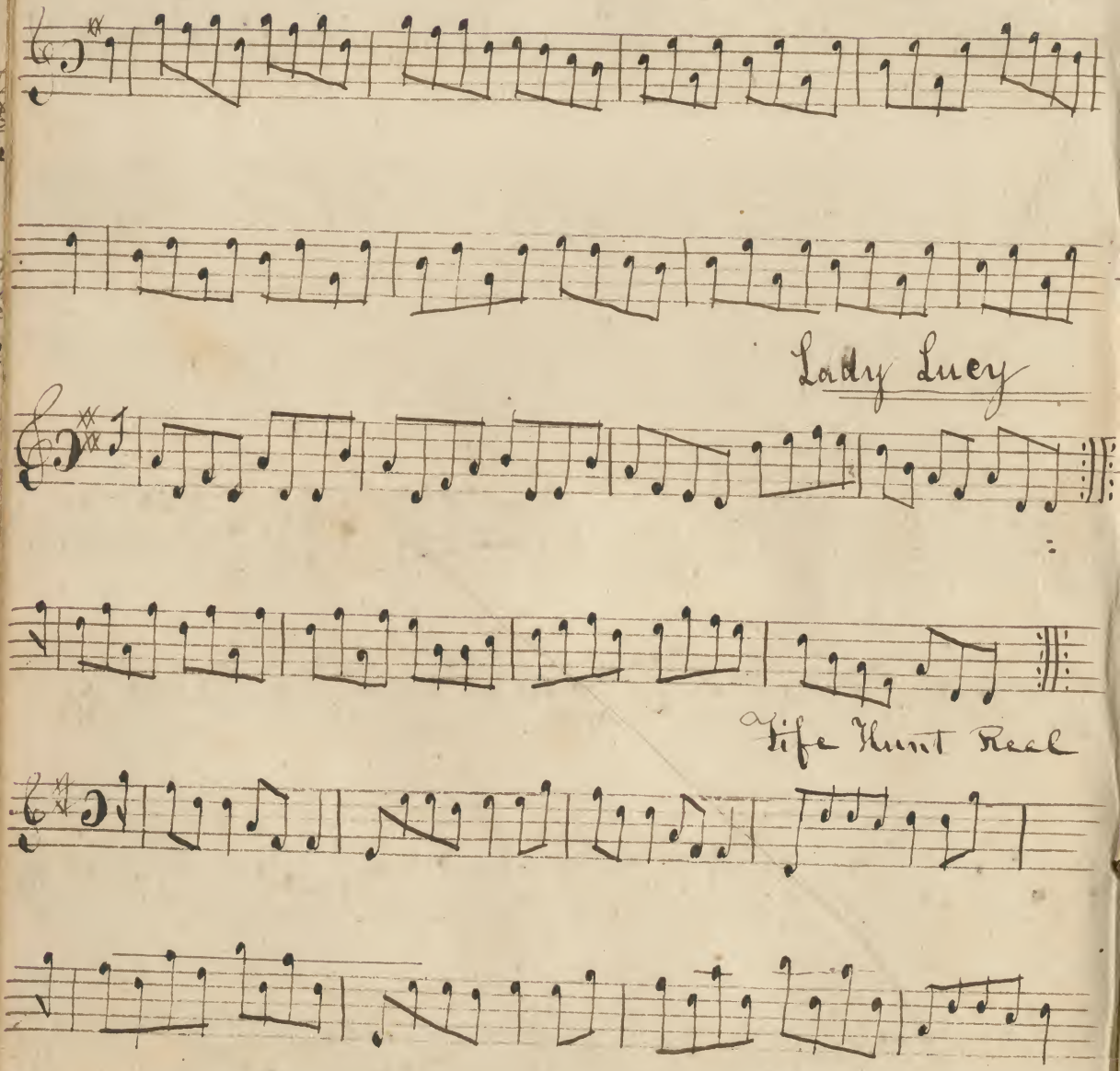
Waltz.



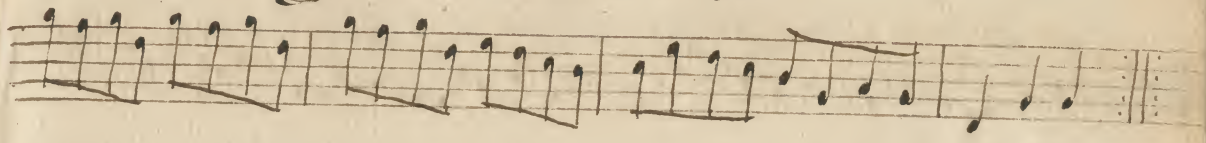
Jigg

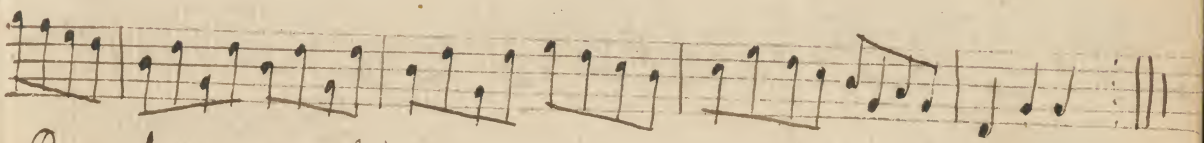


The D— I

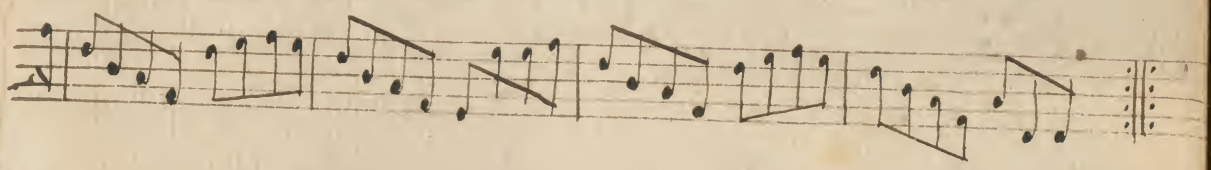


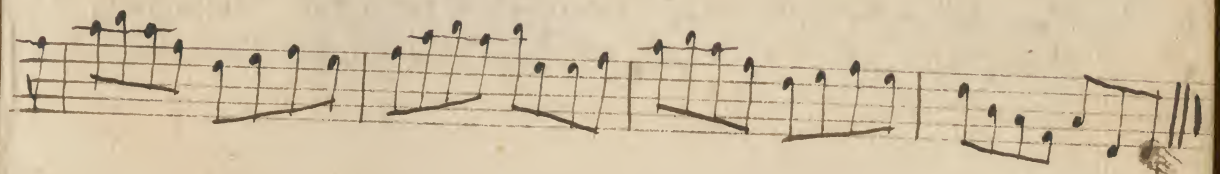
among the Tailors

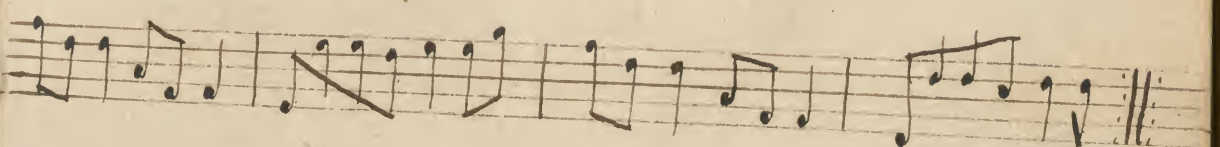


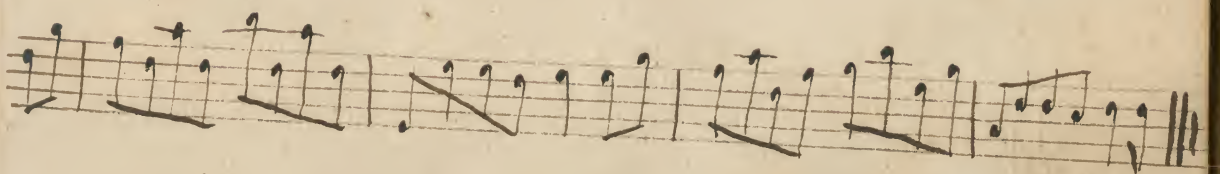


Campbell's Reel



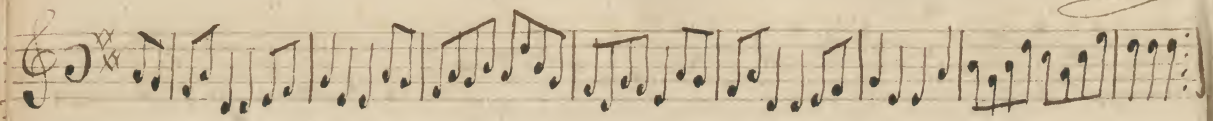






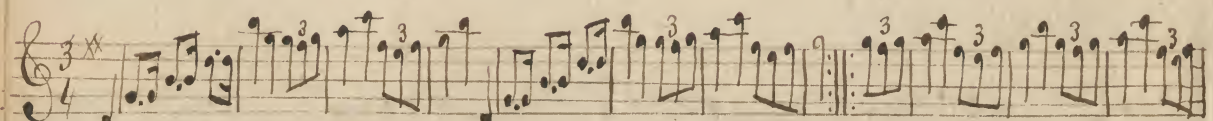
52

New Century

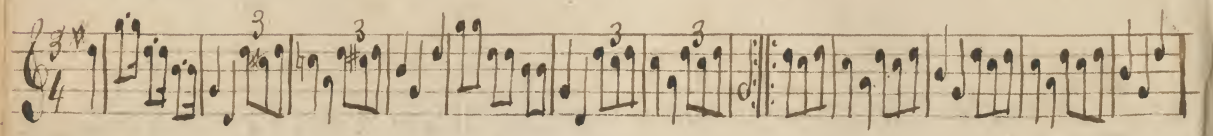


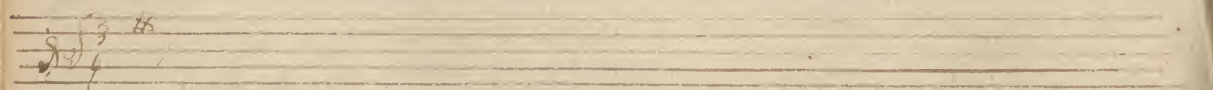
Daddy Carey

Tyrolese

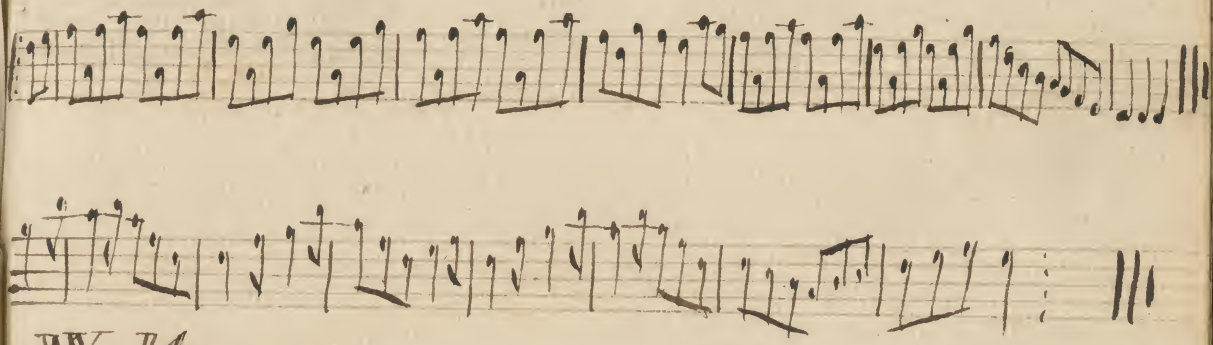


The Anti Tyrlocan

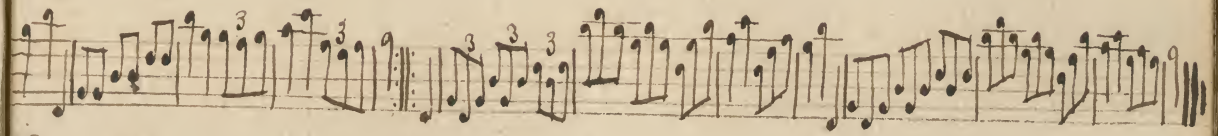




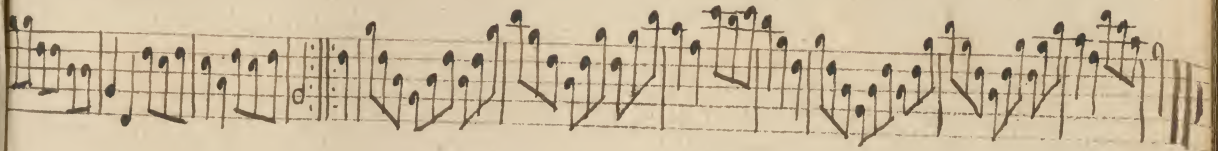
Hornpipe



Waltz.



Waltz.



Reduce $\sqrt{x-a} = \sqrt{x} - \frac{1}{2}\sqrt{a}$. Squaring both sides —

Ex. 10.

$$x - a = x - \sqrt{ax} + \frac{a}{4}$$

Transposing $x - \sqrt{ax} + \frac{a}{4}$

$$\sqrt{ax} = a + \frac{a}{4}$$

Squaring both sides

$$16ax = 25a^2$$

$$x = \frac{25a^2}{16a} = \frac{25a}{16}.$$

11 Ex. Page 129 Day's Algebra. Reduce $\sqrt{5x} \times \sqrt{x+2} = 2 + \sqrt{5x}$. Squaring

Ex. 11.

$$\sqrt{5x+10} = 2 + \sqrt{5x}$$

both sides

$$5x + 10 = 4 + 4\sqrt{5x} + 5x$$

Transp

$$6 = 4\sqrt{5x}$$

Squaring again

$$16 \times 5x = 36 \quad 80x = 36$$

$$x = \frac{36}{80} = \frac{9}{20}.$$

12. Reduce $\frac{x - ax}{\sqrt{x}} = \frac{\sqrt{x}}{x}$

Clear of fractions

$$x^2 - ax^2 = x$$

Dividing by x

$$x - ax = 1$$

Dividing again by x

$1 - a = \frac{1}{x}$. This $\frac{1}{x}$ is the reciprocal of x: and hence, if $1 - a = \frac{1}{x}$ the reciprocal of $1 - a$ is x. Therefore must $x = \frac{1}{1-a}$ the reciprocal of $1 - a$.

Ex. 15. Reduce $x + \sqrt{a^2 + x^2} = \frac{2a^2}{\sqrt{a^2 + x^2}}$

Clear of fract.

$$x\sqrt{a^2 + x^2} + a^2 + x^2 = 2a^2$$

Transp x and $\sqrt{\ }$

$$x\sqrt{a^2 + x^2} = a^2 - x^2$$

Div. 2 mem. by x & Square both sides

$$a^2 + x^2 = a^4 - 2a^2x^2 + x^4$$

Clear of fract. $a^2x^2 + x^4 = a^4 - 2a^2x^2 + x^4$

Transp & Chang. signs

$$3a^2x^2 = a^4$$

$$x^2 = \frac{a^4}{3a^2} = \frac{a^2}{3} = a^2 \times \frac{1}{3}$$

$$x = a\sqrt{\frac{1}{3}}.$$

Ex 14. Reduce $\sqrt{x} + \sqrt{a+x} = \frac{2a}{\sqrt{a+x}}$

Clearing fraction

$$\sqrt{ax + x^2} + a + x = 2a$$

transposing & omitting $\sqrt{\ }$

$$\sqrt{ax + x^2} = a - x$$

Squaring both members

$$ax + x^2 = a^2 - 2ax + x^2$$

Transp & Chang. signs

$$3ax = a^2$$

$$x = \frac{a^2}{3a} = \frac{a}{3} \text{ ans.}$$

Ex. 16. Reduce $x + a = \sqrt{a^2 + x} \cdot \sqrt{b^2 + x^2}$ Squaring both sides

$$x^2 + 2ax + a^2 = a^2 + x\sqrt{b^2 + x^2}$$

Transp a^2 & Div. by x

$$x + 2a = \sqrt{b^2 + x^2}$$

Squaring both sides

$$x^2 + 4ax + 4a^2 = b^2 + x^2$$

Transposing $4a^2 + x^2$

$$4ax = b^2 - 4a^2$$

$$x = \frac{b^2 - 4a^2}{4a} \text{ or } \frac{B^2 - A}{4a}$$

Ex 17. Reduce $\sqrt{2+x} + x = \frac{4}{\sqrt{2+x}}$

Clear of fract.

$$2 + x + x\sqrt{2+x} = 4$$

Transp & Square

$$2x + x^2 = 4 + 4x + x^2$$

Transp again

$$4x - x^2 = 2 - x$$

$$6x = 4$$

$$x = \frac{4}{6} = \frac{2}{3}.$$

18th Ex. Reduce $\sqrt{x-32} = 16-\sqrt{x}$. Squaring both sides.

$x-32 = 256-32\sqrt{x}+x$. Transp. & uniting

$-x \quad 32) \quad 32$

$32\sqrt{x} = 288$ Dividing by 32

288

$\sqrt{x} = 9$

$x = 81.$

~~Square both Members~~

~~Transp. 4x+1~~

Ex 19. Reduce $\sqrt{4x+17} = 2\sqrt{x}+1$

Squaring both sides

Transp. 4x+1

$4x+17 = 4x+4\sqrt{x}+1$

$4x-1$

$0+16$

$2\sqrt{x}+1$

$4\sqrt{x^2}+4\sqrt{x}+1$

Only this term $= 4x$

$4\sqrt{x} = 16$

$\sqrt{x} = \frac{16}{4}$ $x = 4^2 = 16.$

Ex. 21. Reduce $\frac{\sqrt{6x-2}}{\sqrt{6x+2}} = \frac{4\sqrt{6x-9}}{4\sqrt{6x+6}}$

Clearing fractions $4\sqrt{6x-9} \quad \sqrt{6x-2}$

$4\sqrt{36x^2}+8\sqrt{6x}$
$-9\sqrt{6x}-18$

$4\sqrt{36x^2}+6\sqrt{6x}$
$-8\sqrt{6x}-12$

$4\sqrt{36x^2}-\sqrt{6x}-18 = 4\sqrt{36x^2}-2\sqrt{6x}-12$ Cancelling $4\sqrt{36x^2}$ on both sides

$-\sqrt{6x}-18 = 2\sqrt{6x}-12$ Changing all the signs & transp. &c.

$12 \quad \sqrt{6x}$

$6 = \sqrt{6x}$ Squaring both members

$36 = 6x : x = \frac{36}{6} = 6.$

Prob. 8 - pag. 134. What 2 numbers are those whose diff. is to the greater as 2:9 and the difference of whose squares is 128. Let $2x$ = their diff. &

$9x$ = the greater number.

Then will $7x$ = the lesser. But the product of the sum & difference of any two numbers is equal to the diff. of their squares. Hence $(9x+7x) = 9x+7$

$(9x-7x) = 9x-7$

Proof. Substituting 2 in the room of x —

$9\times2 = 18$
$7\times2 = 14$ } Numbers required

$81x^2-49x^2$
$49x^2$
$32x^2$

$81x^2+63x^2$
$-63x^2-49x^2$

$81x^2-49x^2 = 128$
$49x^2$

$32x^2 = 128$

$x^2 = 32 = 4$

$x = \sqrt{4} = 2.$

The product of any number multiplied by 4, is equal to the square of twice the square root of such number. Thus $49\times4 = 196$. and twice the $\sqrt{49} = 14^2 = 196$.

Prob 12 Two Travellers, A & B set out to meet each other — A leaving the Town of C. at the same time B left the Town of D. They travelled the direct road between C & D; and on meeting, it appeared that A had travelled 18 miles more than B, and that A could have gone B's distance in 15 days & 3/4 — but B would have been 28 days in going A's distance —

Required the distance between the Towns.

Let x = the miles A travelled

then $x-18$ = the miles B travel — $\frac{x-18}{15\frac{3}{4}}$ = a day's travel of A

& $\frac{x}{28}$ = B's daily travel

Hence $x : x-18 : \frac{x-18}{15\frac{3}{4}} : \frac{x}{28}$

$\frac{x^2}{28} = \frac{x^2-36x+324}{15\frac{3}{4}}$ Multiplying this equation by 4 to get rid of $\frac{3}{4}$

$\frac{4x^2}{112} = \frac{4x^2-144x+1296}{63}$ Clearing this of fractions

28) $252x^2 = 448x^2 - 16128x + 145152$. Dividing by 28

$9x^2 = 16x^2 - 576x + 5184$ Transp'g & uniting terms

$7x^2 - 576 = -5184$ Completing the square by multiplying the equation by 4 times the Co. eff. of the highest power of the unknown quant. & by adding the square of the Co. eff. of x to both members

28

$196x^2 - 16128x + 331776 = -145152 + 331776$

By extr'g sq. root of this equation we $\left\{ \begin{array}{l} +145152 \\ 186624 \end{array} \right.$

have $14x^2 = 576 + 432$

$$\begin{array}{r} 432 \\ 14\overline{)1008}(72 \\ 98 \\ \hline 28 \end{array}$$

$72 = x$ the distance A travelled, &

18

54 = the distance B travelled

So that 126 miles is the distance between C & D.

Another rule for completing the square of a binomial.

If the highest power of the unknown quantity have a coefficient equal to only One multiply the equation by 4 & add to both sides the square of the coefficient of the lowest power if the Co. eff. of the highest be 2-3-4 &c. Multiply the equation by 8-12-16 &c & add the square of the Coeff. on the first power as before. The root extracted will leave the unknown quantity simply multiplied

Ex. 7 Page 147 Reduce $\frac{x+4}{3} - \frac{7-x}{x-3} = \frac{4x+7}{9} - 1$ Clear fractions by 9

$9x^2+9x-108+27x-189 = 12x^2-15x-63$ Transp & uniting &c

$3x^2-78x = -315$ Divide by 3

$x^2-26x = -105$ Comp. Sqre by add'g $\frac{1}{2}$ Co. of x

$x^2-26x+169 = 169-105 = 64$

$x = 13 \pm \sqrt{64} = +8 = 21$
or $13-8 = -5$

8th Ex. Reduce $\frac{x^3-10x^2+1}{x^2-6x+9} = x-3$ Clear fractions

$x^3-9x^2+27x-27 = x^3-10x^2+1$ $-x^3$ is balanced

Trans & unity terms

$x^2+27x = 28$ Comp. Sqre

$4x^2+108x+729 = 112+729 = 841$ Ext. sq. rt

$2x+27 = \sqrt{841} = 29$

$4 \overline{)841}$ $\quad 29$

$2x = -27+29 = 2$

$x = 29-27 = \frac{2}{2} = 1$ or $29+27 = \frac{56}{2} = 28$

Ex 3 Reduce $4x - \frac{14-x}{x+1} = 14$ $\overset{Q}{=}$ $4x^2+4x-14+x = 14x+14$

$x^2-\frac{9x}{4} = 7$ Comp. Sqre

$x^2-\frac{9x}{4}+\frac{81}{64} = 7+\frac{81}{64} = \frac{529}{64}$

$x = \frac{9}{8} \pm \sqrt{\frac{529}{64}} = \frac{23+9}{8} = \frac{32}{8} = +4$

or $\frac{23}{8} - \frac{9}{8} = \frac{14}{8} = \frac{7}{4}$ or $1\frac{3}{4}$.

Ex 14 Reduce

2) $2\sqrt[3]{x^2}+3\sqrt[3]{x} = 2$ Divide by 2

$x^{\frac{2}{3}}+\frac{3}{2}\sqrt[3]{x}+\frac{9}{16} = 1+\frac{9}{16}$ Comp. Sqre

$x^{\frac{1}{3}} = -\frac{3}{4} \pm \sqrt{1+\frac{9}{16}}$ Ext. sq. rt

$x = \left(-\frac{3}{4} \pm \sqrt{1+\frac{9}{16}}\right)^3$ Involving both sides

The value of the quantity under the radical sign is $\sqrt{\frac{25}{16}} = \frac{5}{4}$: and the sum of $\frac{5}{4} - \frac{3}{4}$ is $\frac{2}{4}$: the Cube of which is $\frac{8}{64}$ or $\frac{1}{8}$ the value of x

Ex 10 Reduce $\frac{3x}{x+2} - \frac{x-1}{6} = x-9$ Clearing fraction

$18x - x^2 - x + 2 = 6x^2 - 42x - 108.$ Transp & uniting &c

$7x^2 - 59x = 110$

28 Comp sqn by multg by 28

$196x^2 - 1652x + 3481 = 3080 + 3481$ Ext root

$14x = 59 \pm \sqrt{6561} = 81$ $\quad +3080$ $\quad 6561$

81

$x = 140 \div 14 = 10$ Ans.

Ex 11. Reduce $\frac{x}{a} + \frac{a}{x} = \frac{2}{a}$ Clearg fractions

a) $ax^2 + a^3 = 2ax$ Dividing by a

$x^2 + a^2 = 2x$ Transp

$x^2 - 2x = -a^2$ Comp sqn

$x^2 - 2x + 1 = 1 - a^2$ Ext root

$x = 1 \pm \sqrt{1-a^2}.$

Ex 9. Reduce $\frac{6}{x+1} + \frac{2}{x} = 3.$ Clearg fract

$6x + 2x + 2 = 3x^2 + 3x$ Trans & unit

$3x^2 - 5x = -2$ multg by 12 & addg sqn of coeff^t = 5

12

$36x^2 - 60x + 25 = 24 + 25 = 49$ Ext. root

$6x = 5 \pm \sqrt{49} = 7$

7

$x = 12 \div 6 = 2.$ Ans.

Ex 18. Reduce $3x^{2n} - 2x^n = 8$

12

$36x^{2n} - 24x^n + 4 = 96 + 4$ Comp sqn

$6x^n = 2 \pm \sqrt{96+4} = 10$ Ext sq. root

10

$6x^n = 12.$

$x = \sqrt[n]{12 \div 6} = \sqrt[n]{2}.$

19th omit the multiplication transpo & change all the signs — and

Ex. 19. Reduce $2(1+x-x^2) - \sqrt{1+x-x^2} = -\frac{1}{9}.$

$x^2 - x + \frac{1}{4} = (+\sqrt{x^2 - x + 1}) = 1 + \frac{1}{9} + \frac{1}{4}$ complete sqn.

$x = \frac{1}{2} + \sqrt{1 + \frac{1}{9} + \frac{1}{4}} = \frac{1}{6}\sqrt{49}.$ Ext sq root. $1 + \frac{1}{9} = \frac{10}{9}$ $\quad \frac{40}{36} = \frac{10}{36}$

But this is not right.

See page 67.

Ex 20. $\sqrt[3]{x^3 - a^3} = x - b$ Cube both sides

$x - b$
$x - b$
$x^2 - bx$
$-bx + b^2$
$x^2 - 2bx + b^2$
$x - b$
$x^3 - 2bx^2 + b^2x$
$-bx^2 + 2b^2x - b^3$

$x^3 + a^3 = x^3 - 3bx^2 + 3b^2x - b^3$ Expung the x^3 terms

(36) $36x^2 - 36b^2x = a^3 - b^3$ Change all the signs transpos $-a^3 + b^3$ and divide by 36

$x^2 - bx = \frac{a^3 - b^3}{36}$

$x^2 - bx + \frac{b^2}{4} = \frac{a^3 - b^3}{36} + \frac{b^2}{4}$ Comp square

$x = \frac{b}{2} + \sqrt{\frac{a^3 - b^3}{36} + \frac{b^2}{4}} =$ Extract

$\frac{\sqrt{4a^3 - 4b^3 + 3b^3}}{12b} = \frac{4a^3 - b^3}{12b}$

Ex. 21. Reduce $\frac{\sqrt{4x+2}}{\sqrt{x}} = \frac{4 - \sqrt{x}}{\sqrt{x}}$

$\sqrt{4x+2} = 4 - \sqrt{x}$

$\sqrt{4x^2 + 2x} = 2x + \sqrt{2x} = 16 - x$ transp x

$3x + \sqrt{2x} = 16$ sq sides

$3x + \sqrt{2x}$
$9x^2 + 3x\sqrt{2x}$
$+3x\sqrt{2x} + 2x$
$9x^2 + 6x\sqrt{2x} + 2x = 256$ Extract

$3x + x = 16$
$4x = 16$
$x = 16 \div 4 = 4$

$\left(\frac{4 - \sqrt{x}}{4 + \sqrt{x}}\right)$
$16 - 4\sqrt{x}$
$+4\sqrt{x} + x$
$16 - x$

Not quite satisfactory

21 Again

$\sqrt{4x^2 + 2x} = 16 - x$ $\sqrt{4x^2} = 2x$
$2x + 2\sqrt{x} = 16 - x$ Remove $2x$ to left side
$2\sqrt{x} = 16 - 3x$
$4x = 256 - 96x + 9x^2$ Trans & unite
$9x^2 - 100x = -256$ Comp square
36
$324x^2 - 3600x + 10000 = -9216 + 10000 = 784$ Extract
$18x = 100 \pm \sqrt{784} = 28$
$x = 100 - 28 = 72 \div 18 = 4$

Ex 13. Reduce $\frac{x^6}{2} - \frac{x^3}{4} = -\frac{1}{32}$

$128x^6 - 64x^3 = -8$
$x^6 - \frac{1}{2}x^3 = -\frac{1}{16}$
$x^6 - \frac{1}{2}x^3 + \frac{1}{16} = -\frac{1}{16} + \frac{1}{16}$
$x^3 = \frac{1}{4}$
$x = \sqrt[3]{\frac{1}{4}}$

Ex 21 again $\frac{\sqrt{4x+2}}{\sqrt{x}} = \frac{4 - \sqrt{x}}{\sqrt{x}}$

$\sqrt{4x^2 + 2x} = 16 - x$
$2x + 2\sqrt{x} = 16 - x$
$2\sqrt{x} = 16 - 3x$
$4x = 256 - 96x + 9x^2$
$9x^2 - 100x = -256$
36
$324x^2 - 3600x + 10000 = 9216 + 10000 = 784$ Extract
$18x = 100 \pm \sqrt{784} = 28$
$x = 100 - 28 = 72 \div 18 = 4$
$x = 4^2 = 4$

66 **Ex 15.** Reduce $\frac{1}{2}x - \frac{1}{3}\sqrt{x} = 22\frac{1}{2}$ Dividing th equ by $\frac{1}{2}$

$\sqrt{x}+\frac{2}{3}$ are the roots of this square

$x - \frac{2}{3}\sqrt{x} = 44\frac{1}{3}$ add $\frac{2}{3}$

$x - \frac{2}{3}\sqrt{x} + \frac{4}{36} = 44\frac{1}{3} + \frac{4}{36}$ Compl squre

$\sqrt{x} = \frac{2}{6} \pm \sqrt{44\frac{1}{3} + \frac{4}{36}}$ Ext $\pm$ r

$x = \frac{2}{6} \pm \sqrt{44\frac{1}{3} + \frac{4}{36}}^2$

$44\frac{1}{3} = 44\frac{3}{9}$

$\frac{4}{36} = \frac{1}{9}$

$44\frac{4}{9} = \frac{400}{9}$ + th

Shorter method.

$\frac{1}{2}x - \frac{1}{3}\sqrt{x} = 22\frac{1}{2}$ Clear fration

$18x - 12\sqrt{x} = 198$. Divid. by 6

$3x - 2\sqrt{x} = 133$ Compl. squa

$36x - 24\sqrt{x} + 4 = 4 + 1596 = 1600$

$6\sqrt{x} = 2 \pm \sqrt{1600} = 40 + 2 = 42$

$\sqrt{x} = 42 \div 6 = 7$

$x = 7^2 = 49$.

Square root of this $\frac{20}{3}$ and th $\frac{1}{3}$ last transposed $= \frac{1}{3}$, which added to $\frac{20}{3} = \frac{21}{3}$ and the square of this is $\frac{441}{9}$ making

$x = 49$.

Ex. 16 Reduce $2x^{\frac{1}{2}} - x^2 + 96 = 99$. Transp +96 $99 - 96 = 3$

$16x^2 - 8x^2 + 1 = 1 + 24$ Compl squa by multg by 8 times Co

$4x^2 = 1 \pm \sqrt{1+24} = 25 = 5$. Ext sq root

$x^2 = 1 + 5 = 6$

$x = \sqrt{\frac{6}{4}} = \frac{1}{2}\sqrt{6}$.

Ex 17 – Reduce $\sqrt{(10+x)} - (10+x)^{\frac{1}{4}} = 2$. Reduce it a com index to x

Compl squa

$\sqrt[4]{10+x}^2 - \sqrt[4]{10+x} + \frac{1}{4} = 2 + \frac{1}{4}$

Then $\sqrt{10+x}^2 = \frac{1}{2} \pm \sqrt{2+\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$. Sq both sides

$10 + x^2 = 4$.

$x = 4^2 = 16 - 10 = 6$. Ans.

The $\frac{1}{4}$ root of $10+x$ subtracted from the square root of the same quantity, leaves the $\frac{1}{4}$ root of that quantity: The $\sqrt[4]{10+x} = 2$.

$10+x$ therefore is equal to $2^4 = 16$ and $x = 16 - 10 = 6$. Answ

Ex. 18. Reduce $3x^{2n} - 2x^n = \frac{8}{12}$

$36x^{2n} - 24 + 4 = 96 + 4 = 100$

$6x^n = 2 \pm \sqrt{100} = 10 + 2 = 12$ Ext r

$x^n = \sqrt[n]{\frac{12}{6}} = \sqrt[n]{2}$.

Ex. 22. Reduce $x^{\frac{6}{5}}+x^{\frac{3}{5}}=756$

$x^{\frac{6}{5}}+x^{\frac{3}{5}}+\frac{1}{4}=756+\frac{1}{4}$ Comp. of square

$x^{\frac{3}{5}}=-\frac{1}{2}\pm\sqrt{756+\frac{1}{4}}=$ Ext. root

$756+\frac{1}{4}$
4
$3025 \mid 55 \div \sqrt{4}=2=27+\frac{1}{2}$
25 $\quad -\frac{1}{2}$
$105 \mid 525$
525
$\ldots$

~~Involving~~ — $x^{\frac{1}{5}}=\sqrt[3]{27}=3$

Involving $x^{\frac{2}{5}}=9$

$x^{\frac{3}{5}}=27$

$x^{\frac{2}{5}}\times x^{\frac{3}{5}}=x^{\frac{5}{5}}$ or $x=243$. Ans.

$x^{\frac{3}{5}}=27$ and

$x^{\frac{3}{5}}\times x^{\frac{3}{5}}=x^{\frac{6}{5}}=729=27\times 27$

756

Ex. 19. Reduce $2(1+x-x^2)-\sqrt{1+x-x^2}=-\frac{1}{9}$. Squaring $\sqrt{1+x-x^2}$ & subtracting it

Mult. by 2 — $2+2x-2x^2$

Subtracting — $+1+x-x^2$

$1+x-x^2=-\frac{1}{9}$.

See again next page.

Chang. signs & transp. $x^2-x=-\frac{1}{9}+1$

Comp. of square $x^2-x+\frac{1}{4}=\sqrt{1+\frac{1}{4}-\frac{1}{9}}=\frac{5}{4}-\frac{1}{9}$

Ext. root — $x=\frac{1}{2}\pm\frac{1}{6}\sqrt{41}$. $\quad \frac{45-4}{36}=\sqrt{\frac{41}{36}}=\frac{1}{6}\sqrt{41}$

It is not perceived why $-\frac{1}{9}$ should not be changed to $+\frac{1}{9}$; nor how the above value of x can be found without subtracting $-\frac{1}{9}$ from $1+\frac{1}{4}$.

Ex. 23. Reduce $\sqrt{2x+1}+2\sqrt{x}=\frac{21}{\sqrt{2x+1}}$. Clear. fractions

$2x+1+2\sqrt{x^2+2\sqrt{x}}=21$. transp. $2x+1$

$2\sqrt{2x^2+2\sqrt{x}}=20-2x$

Introducing the Coeff. of $\sqrt{x^2}$ under the rad. sign — $\sqrt{8x^2+4x}=20-2x$ Squar. both sides

$20-2x$
$400-40x$
$-40x+4x^2$

$8x^2+4x=400-80x+4x^2$ Transp. & unity

4) $4x^2+84x=400$ Dividing by 4

$x^2+21x=100$ Compl. square

$x^2+21x+110.25=100+110.25$

$x=-10.5\pm\sqrt{210.25}$

$+14.5$
$+4$ or
-25

100
$210.25 \mid 14.5-10.5=+4$.
1
$24 \mid 110$
$4 \quad 96$
$285 \mid 1425$
$5 \quad 1425$
$\ldots$

$\sqrt{2x+1}$
$2\sqrt{x}$
$2\sqrt{2x^2+2\sqrt{x}}$
$=2\sqrt{2x^2+4x}$

* Twice the square root of any quantity, is equal to the square root of 4 times such quantity.

68 Ex. 24. Reduce $2\sqrt{x-a}+3\sqrt{2x}=\frac{7a+5x}{\sqrt{x-a}}$ Chang fracts

Transp $2x-2a$ & Dividing by 3 - and Squaring both sides

$$2\sqrt{x^2-a^2}+3\sqrt{2x^2-2ax}=7a+5x$$
$$+2a-2x$$
$$3\overline{)9a-3x}$$
$$3a-x$$
$$3a-x$$
$$9a^2-3ax$$
$$-3ax+x^2$$
$$2x^2-2ax=9a^2-6ax+x^2$$
$$-x^2-6ax$$
$$x^2-8ax=9a^2 \text{ Comp. square}$$
$$x^2-8ax+16a^2=9a^2+16a^2=25a^2$$
$$x=4a\pm\sqrt{25a^2}\ \ 5a=+9a$$
$$25a^2 \quad a-a.$$

Ex 25. Reduce $x+16-7\sqrt{x+16}=10-4\sqrt{x+16}$ transp $-4\sqrt{x+16}$

$$+4\sqrt{x+16}$$
$$x+16-3\sqrt{x+16}=10 \text{ putting } -3 \text{ under rad. sign}$$
$$\sqrt{9x+144}=-x-6. \text{ \& transp } x+16$$
$$-x-6 \text{ squaring both sides}$$
$$x^2+6x$$
$$+6x+36$$
$$9x+144=x^2+12x+36 \text{ Transp \& unity}$$
$$x^2+3x=108 \text{ Complete square}$$
$$4$$
$$4x^2+12x+9=432+9$$
$$2x=-3\pm\sqrt{441}\ \ 21-3=18\div 2=+9$$
$$4 \quad 41\overline{)41}\ \ 41 \quad \text{Ext. root}$$
$$\text{or } -12$$

Ex 13. Reduce $\frac{x^6}{2}-\frac{x^3}{4}=-\frac{1}{32}$

$$128x^6-64x^3=-8$$
$$512$$
$$65536x^6-4x+4096=-4096 \text{ Compl square}$$
$$256x^3=64 \text{ Ext root}$$
$$x^3=64\div 256=\frac{1}{4}$$
$$x=\sqrt[3]{\frac{1}{4}}.$$

Ex-19 again Reduce $2(1+x-x^2)-\sqrt{1+x-x^2}=-\frac{1}{9}$ Complete square by mult by 8

$$8$$
$$16(1+x-x^2-8\sqrt{1+x-x^2}+1=\frac{8}{9}+1=+\frac{1}{9}$$
$$4\sqrt{1+x-x^2}-1=\frac{1}{3} \text{ Transp } -1 \text{ \& dividing by 4.}$$
$$\sqrt{1+x-x^2}=\frac{1}{3}=1+\frac{1}{3}\div 4 \text{ - squaring both sides}$$
$$1+x-x^2=\frac{1}{9} \text{ Transp \& changing signs}$$
$$x^2-x=1-\frac{1}{9}=+\frac{8}{9}$$
$$x^2-x+\frac{1}{4}=\frac{1}{4}+\frac{8}{9}=\frac{41}{36} \text{ Comp square}$$
$$x=\frac{1}{2}\pm\sqrt{\frac{41}{36}}=\frac{1}{6}\sqrt{41}.$$
$$x=\frac{1}{2}+\frac{1}{6}\sqrt{41}.$$

27 Ex. Reduce $\frac{4x-5}{x} - \frac{3x-7}{3x+7} = \frac{9x+23}{13x}$

$39x^2+91x$
$4x-5$
$156x^3+169x^2-455x$

$156x^3+169x^2-455x-39x^3+91x = 27x^3+132x^2+161x$
$-66x^3+91x^2-161 \quad -27x^3$
$+260x^2 \quad -66x^3$
132
$90x^3+128x^2-616x$

$-3x+7$
$13x^2$
$-39x^3+91x$

Divide by x) $90x^3+128x^2-616x$
2) $90x^2+128x=616$
$45^2+64x=308$ Completing square
180

$9x+23$
$3x^2+7x$
$27x^3+132x^2+161x$

$8100x^2+11520x+4096=55440+4096=59536$ Extg root

$90x=-64\pm\sqrt{59536}$ | 244
4 | -64
44 | 195
4 | 176
484 | 1936
4 | 1936

$180\div 90=2$ the value of x.

Ex. 28. Reduce $\frac{3}{6x-x^2}+\frac{6}{x^2+2x}=\frac{11}{5x}$.

$6x-x^2$
$5x$
$30x^2-5x^3$
6
$180x^2-30x^3$

x^2+2x
$5x$
$5x^3+10x^2$
3
$15x^3+30x^2$

$15x^3+30x^2-30x^3+180x^2=-11x^4+44x^3+132x^2$

$11x^4-59x=-78$ Compl. square Transp. & unity & divide by x^2 & change signs
44
$484x^2 \quad +3481=-3432+3481$
3432
$\sqrt{49}$ | Ext root
49 | 7

$22x=59+7=66\div$
$x=66\div 22=3.$

$6x-x^2$
x^2+2x
$6x^3-x^4+12x^2$
$-2x^3$
$4x^3-x^4+12x^2$
11
$44x^3-11x^4+132x^2$

Ex 29. Reduce $(x-5)^3-3(x-5)^{\frac{3}{2}}=40$ Completing the square
4
$4(x-5)^3-12(x-5)^{\frac{3}{2}}+9=160+9$ Extg root.
$2\sqrt{x-5^3}=3\pm\sqrt{169}=13$
$2\sqrt{x-5^3}=16$
$\sqrt{x-5^3}=16\div 2=8$ squaring both sides
$x-5^3=64$
$x=\sqrt[3]{64}+5=4+5=9.$

Ex. 30. Reduce $x+\sqrt{x+6}=2+3\sqrt{x+6}$ Transposing 2 & $\sqrt{x+6}$.
$x-2=2\sqrt{x+6}$ squaring both sides
$x^2-4x+4=4x+24$ Transp. 4 & $4x$
$x^2-8x=20$ Compl. square
$x^2-8x+16=20+16=36$
$x=4\pm\sqrt{36}=+6=10.$

Prob 10 – 150 A Gent. bought a number of pieces of Cloth for 675 Dollars, which he sold again at 48 Dollars a piece, & gained by the bargain as much as one piece cost him.

What was the number of pieces?

Let x = the number of pieces –

Then $\frac{675}{x}$ = the price per piece at which the pieces were bought –

And $48x$ = what all pieces were sold for – By the conditions of the problem – all the pieces were sold for as much more than they cost, as one piece cost. Hence

$$48x - 675 = \frac{675}{x}$$

$$48x^2 - 675x = 675 \quad \text{Completing the square}$$

192 192

96	1350	675
432	6075	675
48	675	3375
		4725
		4050
9216	$6x = 129600x + 1/2 = 129600 + 455625$	455625
81	96x	

186 | 1116
6 | 1116

455625

585225 | 765
49
146 | 952
6 | 876
1525 | 7625
5 | 7625

$96x = 675 + 765$
765
$x = 1440 \div 96 = 15$ the number of pieces. Ans.
96
480
480

Prob 15. Several Gent. ran up a bill to 175 Dollars. When 2 of them having absquatilated – the bill was paid by the others: Each one contributing two Dollars more, than would have been his share, had the bill been paid by the whole Comp. What was the number of the Comp at first? Let x = the number

$$\frac{175}{x} + 10 = \frac{175}{x-2}$$

$175x - 350 + 10x^2 - 20x = 175x$ Expunge $175x$: then

$x^2 - 2x = 35$ transpose –

$x^2 - 2x + = 35 + 1$

$x = 1 \pm \sqrt{36} = 6 + 1 = 7$. the nº Comp at first –

Prob. 24 - A Gent. bought a certain number of Tal Oxen for 80 Guineas. If he had bought 4 more for the same money, he would have paid one Guinea less for each. What number did he buy? Let x = the number

Then $\frac{80}{x}$ = the price of each: But $\frac{80}{x+4}+1=\frac{80}{x}$

Expng $80x$ — $80x+x^2+4x=80x+320$

$x^2+4x=320$

$x^2+4x+4=324$ Comp. Square

$x=-2\pm\sqrt{324}$ | $18-2=16$. Ans.

$\begin{array}{r|l} 28 & 224 \\ 8 & 224 \end{array}$

Prob 18. A Gent. bought two pieces of Cloth the finer of which cost 4/ a yd more than the other. The finer piece cost 18£; but the coarser one which was 2 yards longer than the finer cost only 16£. How many yds were there in each piece, & what was the price of a yd of each?

Let x = No yds of the finer — then $x+2$ = No yds of the coarser

And $\frac{18}{x}+\frac{16}{x+2}$ will equal the prices of a yd of each — If each of these fractions be severally multiplied by their respective denominators the sum of their products will be equal to the cost of both pieces.

But $\frac{18x}{x}$ makes x equal to 18; whereas $\frac{16x+32}{x+2}$ makes x equal to 16.

Hence $\frac{18x}{x}+\frac{16x+32}{x+2}+2=2x$. Clearing fractions

Dividing by x and

$18x^2+36x+16x^2+32x+2x^2+4x=2x^3+4x$ Transfg & uniting

$2x^2-32x=72$ Dividg by 2

$x^2-16x=36$ Complg square

$x^2-16x+64=36+64=100$

$x=8\pm\sqrt{100}=10+8=18$ No yds of finer — &

20 - do coarser

Ex. 18 There are two numbers whose product is 135 And the diff. of their squares, is to the square of their diff. as 4 : 1 — What are the numbers? —

Let x & y be the numbers —

1		$xy = 135$
2	— —	$x^2 - y^2 : x - y :: 4 : 1$
3 Expanding		$x^2 - y^2 : x^2 - 2xy + y^2 :: 4 : 1$
Subtract 4 Consequents	—	$2xy - 2y^2 : x - y^2 :: 3 : 1$
5 Dividing by $x - y$	—	$2y : x - y :: 3 : 1$
6 Mult & Expt & means		$2y = 3x - 3y$
7 Transp	—	$5y = 3x$
8 Given $xy = 135$	—	$x = \frac{5y}{3}$
9 Make 2 last equal		$x = \frac{135}{y}$
10 Subst 9 in the last. $x = \frac{135}{9} = 15$		$\frac{135}{y} = \frac{5y}{3} = 5y^2 = 405$ $y^2 = 405 \div 5 = 81$ $y = \sqrt{81} = 9$.

Prob. 13. In a mixture of rum & brandy, the diff. between the quantities, is to the quantity of brandy, as 100 is to the number of Gall^s of rum; & the same diff. is to the quantity of rum, as 4 to the number of gall^s of brandy.

How many Gallons are there of each?

Let x & y represent the quantities. Then by the condition

1st	$x - y : y :: 100 : x$	by inverting the means. $x - y : 100 :: y : x$
2d	$x - y : x :: 4 : y$	by inverting extremes $\frac{4}{x} = \frac{x}{y} :: y : x$
3	$x - y : 100 :: y : x$.	
4	$4 : x - y :: y : x$	
5	$x - y : 100 :: 4 : x - y$.	making 3d & 4th equal
6	$x^2 - y^2 = 400$	Mult Expt & Means
7	$x - y = 20$	Extg root
8	$x = 20 + y$	
9	Subst 20 & $20 + y$ in the 4th probation	$4 : 20 :: y : 20 + y$
10	$80 + 4y = 20y$	Subst 5 in lieu of y in the 8th equation $x = 20 + 5$
11	$80 = 16y$ & $y = 80 \div 16 = 5$.	$y = 5 =$ the brandy & $x = 25$ rum

Prob. 16 There are two numbers, which are to each other, in the duplicate ratio of 4 to 3; and 24 is a mean proportional between them. What are the numbers? Let x & y represent them

Then $x : y :: 4^2 : 3^2 :: 16 : 9$ Multiplying Extremes & means

and $x : 24 :: 24 : y$.

$9x = 16y$ Subst. the value of x in the 2d proportion

$x = \frac{16y}{9}$

$\frac{16y}{9} : 24 :: 24 : y$

$\frac{16y^2}{9} = 576$ clearing fractions

$16y^2 = 5184$

$y^2 = 5184 \div 16 = 324$

$y = \sqrt{324} = 18$ and of course $x = 576 \div 18 = 32$.

There are 4 numbers in Geometrical progression, the second of which is less than the 4th by 24 and the sum of the extremes is to the sum of the means as 7 to 3. What are the numbers

Let x = the 1st term & y = the ratio — Then, by Art. 436

Divid by x) $x : xy :: xy^2 : xy^3$

2 — $1 : y :: y^2 : y^3$ — Substituting $y+24$ agreeably to the conditions of the question

3 — $1 : y :: y^2 : y+24$

4 — $25+y : y^2+y :: 7 : 3$ Sum of Ext. is to sum of means as 7 : 3

5 — $25-y^2 : y^2+y :: 4 : 3$ Subtracting consequents from antecedents

6 — $25-2y^2-y : y^2+y :: 1 : 3$ Subtracting again

7 — $75-6y^2-3y = y^2+y$ Multiplying Extremes & means

8 — $7y^2+4y = 75$ Transposing & uniting terms

28

600 Compl. square

150

$196y^2+112y+16 = 2100+16$

$14y = -4 \pm \sqrt{2116} = 46-4 = 42$ [redacted]

And $y = 42 \div 14 = 3$. Substituting 3 in lieu of y in the 3d proportion

$1 : 3 :: 3^2 : 3^3 :: 9 : 27$.

1.3.9.27 the numbers sought.

Last problem in Geomet. progression.

74 Prob. 19 - page 152. A Merchant bought 54 Gallons of Madeira wine, & a quantity of Teneriffe. For the Madeira he paid ½ as many shillings by the Gallon, as there were gallons of the Teneriffe: & for the Teneriffe, he paid 4/ a Gallon less. He sold the mixture for 10/ a Gallon, & lost 28 pounds 16 shillings by his bargain. Required the price of the Madeira, & the number of Gallons of the Teneriffe.

Let x = the number of Galls of Teneriffe - then $\frac{x}{2}$ will represent the price of a gall. of Madeira: and this multiplied by 54 will equal the cost of the Madeira $= \frac{54x}{2} = 27x$

And $\frac{x}{2} - 4$, will represent the price of a gall of Teneriffe; which multiplied by x will equal the cost of the Teneriffe. $\frac{x}{2} - 4 \times$ by $x = \frac{x^2}{2} - 4x$ $\quad \frac{x^2}{2} - 4x$

Whole cost $= \frac{x^2}{2} + 23x$

But the whole quantity was sold at 10/ a gallon

Whole quantity $= 54 + x \times$ by 10 - and the loss 28 : 16

$$\begin{array}{r} 54 + x \\ 10 \\ \hline 540 + 10x \end{array}$$

added $= 576$ shillings $= \frac{x^2}{2} + 23x$

$1116 + 10x = \frac{x^2}{2} + 23x$ Transposing & uniting

$$\frac{x^2}{2} + 23x = 10x + 1116$$

$$x^2 + 46x = 20x + 2232$$

$$\begin{array}{r} -20 \\ \hline x^2 + 26x = 2232 \end{array}$$

$$x^2 + 26x + 169 = 2232 + 169$$

$$\begin{array}{r} 169 \\ \hline 2401 \end{array}$$

$x = -13 \pm \sqrt{2401}$ (49 - 13 = 36 the value of x. $\frac{36}{2}$ = the price of a Gall. Madeira

$$\begin{array}{r} 16 \\ \hline 801 \\ 801 \\ \hline \end{array}$$

89 | 801
9 | 801

& 14 shillings - of a gall Teneriffe

$$\begin{array}{r} 13 \\ 13 \\ \hline 39 \\ 13 \\ \hline 169 \end{array}$$

Prob. 20. page 152.

If the square of a certain number be taken from 40, & the square root of this difference be increased by 10, & the sum be multiplied by 2, & the product divided by the number itself, the quotient will be 4. What is the number?

Let x = the number — then by the conditions $\frac{2\sqrt{40-x^2}+20}{x}=4$

$2\sqrt{40-x^2}+20=4x$ Transposing 20 & dividing by 2

$\sqrt{40-x^2}=2x-10$ Squaring both sides

$40-x^2=4x^2-40x+100$ Transp. & uniting

$5)\ 5x^2-40x=-60$ Dividing by 5

$x^2-8x=-12$ Compt. square

$x^2-8x+16=-12+16=+4$

$x=4\pm\sqrt{4}=2+4=6.$

22. Two casks of wine were bought for $58 Dolls. One of which contained 5 gallons more than the other & the price by the gallon, was $2 less than $\frac{1}{3}$ of the number of gallons in the smaller cask. How many gallons were there in each cask, & what the price per gall.?

Let $x+5$ = the number gall. in the larger cask. & x = the smaller. Then $\frac{x}{3}-2$ = the price of a gallon; & this multiplied into $x+5$ & into x produces $\frac{2x^2}{3}+\frac{5x}{3}-4x-10$ And this = $58.

The greater cask $=12+5=17$

the other $=12$

$\frac{12}{3}-2=$ $2 Doll. a gallon

$\frac{2x^2}{3}+\frac{5x}{3}-4x-10=58$ Clearing fractions — transp. & uniting terms

$6x^2-21x=612$

24

$144x^2-504x+441=14688+441$ Compliting square

441

$12x=21\pm$ 15129 | 123+21=144

$x=144\div12=12$

$\frac{22}{2}$ | 51 / 44 / 720

243 | 729 / 729

76 Prob. 23. In a parcel which contains 24 coins of silver & copper; each silver coin is worth as many cents as there are copper coins; and each copper copper coin is worth as many cents, as there are silver coin: and the whole are worth $2.16 — How many are there of each?

If x = silver coin
$24 - x$ = the copper

$24 - x \times x$
x
$24x - x^2$ = the value of sil. coin
$24x - x^2$ = also, the value of the cop. coin

$48x - 2x^2 = 216$ Chang. signs transp. &c

2) $2x^2 - 48x = -216$
$x^2 - 24x = -108$ Compl'g square
4
$4x^2 - 96x + 576 = 432 + 576$
432
$2x = 24 \pm 12$ $\sqrt{144}$ Ext. root
12
$x = 36 \div 2 = 18$ sil. & 6 cop. coin

Prob 24 — A person bought a certain number of oxen for 80 guineas. If he had bought 4 more for the same money, he would have paid a guinea a head less — What was the number?

Let x = the number. By the conditions

$\frac{80}{x+4} = \frac{80}{x} - 1$ — Clearing fractions
$80x = 80x + 320 - x^2 + 4x$ Transp. & unity terms
$-x^2 + 84x + 320 = 80x$ Changing all the signs
$x^2 - 4x = 320$ Completing square
$x^2 - 4x + 4 = 320 + 4$
$x = 2 \pm \sqrt{324}$ (18
1
28 | 224
224
$x = 2 \pm 18 = +20$
or -16 this is the right value of x

16 oxen for $80 = $5 a head
20 do — 80 = $4 —

Prob. 25 – page 164

There is a certain number consisting of 2 digits. The left hand digit is equal to 3 times the right hand digit; and if 12 be subtracted from the number itself, the remainder will be equal to the square of the left hand digit. What is the number

Let x = the left hand digit & y the right hand digit – note as the local value of figures increases in a ten fold ratio from right to left – the number required $= 10x+y$

By the conditions of the problem — $x = 3y - 1$ — 1

and $10x+y-12 = x^2$ — 2

adding the 2 Eq. $10x+4y-12 = x^2+x$ — 3

Multg 2d by 4 — $40x+4y-48 = 4x^2$ — 4

Subt 3d from 4th — $30x - -36 = 3x^2 - x$ — 5

Transp & uniting $3x^2 - 31x = -36$ — 6

Comp the sq. $36x^2 - 372x + 961 = 432 + 961$ (−432)

Ext root $6x = 31 + 23 \div 6$ ($\pm\sqrt{529}$ (23

$6\overline{)54}$ the left — $\begin{array}{r} 43 \\ 3 \end{array}\overline{)129}$ $\begin{array}{r} 129 \\ \end{array}$

$x = 9$

$x+y = 93$ – hand digit – and $y = 3$

Prob. 26 page 165.

If a certain number be divided by the product of its 2 digits, the quotient will be 2. And if 27 be added to the number, the digits will be inverted. What is the number?

Let $x+y$ = the digits – then by the condition of the problem $\frac{10x+y}{xy} = 2 = 10x+y = 2xy$ and

$10x+y+27 = 10y+x$ — 2

Subtg 1st from 2d — $27 = 10y+x-2xy$ — 3

Transp — $2xy - 27 = 10y+x$ — 4

By the 2d Eq $x = 9y \pm 27$ and substituting this value for x in the 4th Eq. $2xy+27 = 11y-3$ — 5

By last Eq — $x = \frac{11y-3-27}{2y}$ — 6

By makg 2d & 6th equal $\frac{11y-3-27}{2y} = 9y-27$ — 7

Clear fraction $99y-27-243 = 18y^2-54y$ — 8

* The value of the fraction $= y-3$

$\frac{1}{3}$ $18y^2 - 153y = -270$ = 8th transf & multiplied

$6y^2 - 51y = -90$ Comp sq

$144y^2 - 1224y + 2601 = -2160 + 2601$ (2160)

$12y = 51 + 21$ (21)

$y = \frac{72}{12} = 6$. Substituting 6 in 1st Eq. x is found to be equal to 3. So the number sought is 36.

78 Rot 30. Page 165.

Divide 90 into 4 such parts that the 1st +2, the 2d −2, the 3d × by 2, & the fourth divided by 2, shall all be equal.

Let x, y, z be three of the parts & 90 − x − y − z the fourth.

By the conditions $x+2=y-2$

$x = 18 + 2 = 20$
$y = 22 - 2 = 20$
$z = 10 \times 2 = 20$
$w = 40 \div 2 = 20$
90

2 — $x+2 = 2z$
3 — $2z = \frac{90-x-y-z}{2}$ } clearing fractions
4 — $5z = 90 - x - y$ } & transp. — z
5 — $4z = x + y$ multg. 2d Eq by 2. Or, addg together both members of 1st Equation & making the sum = 4z
6 — $9z = 90$ addg 2 last
7 — $z = 10$ from which all the others are easily found.

Jan. 1845 Philadelphia Paper

A Lady being asked her age — ans. that the number expressing it consisted of two figures or digits — the left hand one of which is 5 less than the right hand one: and that 4/7 of the number is less than the product of the two digits by twice the left hand digit. What is her age? Let x equal the left hand & y the right hand digit.

1st Then $x = y - 5$ But the locality of x gives it a tenfold value

Hence the number sought $= 10x + y$ and by the conditions 4/7 of $(10x+y) + 2x = 7xy$

that is $4(10x+y) + 2x = 7xy$

2d Expanded $40x + 4y + 14x = 7xy$ Multg by 7x
3d $7x^2 = 7xy - 35x$
4th $7x^2 - 54x - 4y = -35x$ — Subt 2d from 3d
5 $+4y - 20 = +4x$ mult 1st Eq. by 4
6 — $7x^2 - 54x - 20 = -31x$ add 4 & 5
$7x^2 - 23x = 20$ — — Transpg & multg
28
$196x^2 - 644x + 529 = 560 + 529$ — Compg square
560 Extg root

$14x = 23 +$
33
$x = 56 \div 14 = 4$
and $y = 4 + 5 = 9$ So 49 is the number sought.

1089 | 33
9
63 | 189
3 | 189

A Gentleman sent his Servant with $100 Dollars to purchase Cows at $10 a head. Sheep at $1 Dollar a head & fowls at One Shilling a head so that the number of Animals bought should be equal to the number of Dollars paid for them. What were the numbers of each sort?

It is obvious that the sheep, costing a dollar a head, may be left out of the investigation: And then the question resolves itself into this — "What 2 numbers are those, whose sum is equal to the product of one of them, multiplied by 10, added to the quotient of the other, divided by 8."? Let x = No. Cows & y = No. fowls

Then $x+y = 10x + \frac{y}{8}$ according to the conditions of the question.

Clearing fractions

$8x + 8y = 80x + y$ Transp. & unitiz.

$7y = 72x$ — Here the coefficients designate the numbers sought

That is

7	Cows a $10 =	$70 And
72	fowls a a 1/	9
79		79
21	Sheep of Course	21
100	—	100

So if the Cows had been bought at $15 a head & the fowls at 2/ on the same conditions

Then $x+y = 15x + \frac{y}{4}$

$4x+4y = 60x+y$

$3y = 56x$

Here the No. Cows is $3 \times 15 = 45$ and of fowls $56 \div 4 = 14$

Animals $59 = 59$ Dolls

Corollary — If any unknown quantity be multiplied by any number whatsoever, & be made or found equal to any other unknown quantity, also multiplied by any other number whatsoever — Then, universally, the value of the latter quantity will be equal to the multiplier or coefficient of the former quantity: And the value of the former quantity will be equal to the multiplier or coefficient of the latter: Or the quantities will be equal multiples of those respectively coefficients. Thus if $x = 5y$ then $x = 5$ & $y = 1$, if $9x = 11y$ — $x = 11$ & $y = 9$ Or in the 1st Ex. if y had equalled 11, then x would have = 55

It is plain that if $x = 2, 5, 10$ or $1000y$: Then y will be equal to 2, 5, 10 or a 1000; & y = to 1

From the conditions of a problem, it can readily be determined whether the coefficients be the true numbers sought; or whether the true numbers be some equal multiples of them — Thus in the 7th equation Ex. 10 — page 72, where $3x = 5y$ & where the conditions require that the product of $xy = 135$ &c

Multy by 3

$9x = 15y$

$225 - 81 : 36 :: 4 : 1$

$144 : 36 :: 4 : 1$

gives the true numbers — for $9 \times 15 = 135$

March 21st 1844.

Casuistry

Allow, that conscience rightly regulated;
Implies, conformity to rules, well stated:
That every duty, scrupulously to fulfil,
Claims strict obedience, to the enlightn'd will:
Then, must it not unquestionably follow,
That what's hereto opposed, must clearly needs be hollow?

Suppose obedience challeng'd to a strict command,
That we should neither sit, lie, walk, nor stand;
Or, the performance, of some mental function
Should be enjoin'd, where clearly the injunction
Invades the undoubted province of the will:
Can it be urg'd, duty claims obedience still?

Then to work hard, & not grow faint nor weary,
To endure most wanton wrongs, & yet feel cheery,
To love what's hateful, or to eschew what's good,
Are, or may become duties, not to be withstood;
Provided, an ordinance, be therefor assign'd:
Mooted divine or papal, to sooth the pliant mind.
Commands, like these, so abhorrent to our natures!
Can they ought else be, than very small potatoes?

Suppose again our firm belief's enjoin'd
Of something, not prov'd, nor can be, nor defin'd:
How, what's called God, in times, now grown remote,
Familiar was with man, & to him spake & wrote:
Once, in particular, bared his backside to Mose,
Hid in a cleft, as by it pass'd, he goes.

And often, every now & then, would stray,
To visit man; eat kid, & spend an holliday.
not, that all men alike, his kindly visits shared:
On some he smiled; on others fiercely glared.
Just merely by this means or that, to indicate
That God, like man can love as well as hate.

But rather, cherishing a long pondered notion
That his true glory turn'd on man's devotion:
And that, could he but find a nodel man or nation
It would insure that glory thro' his wide creation.
Thence far & near looked round & spied in Carran
A man called Abram him whose wife was barren.

This man seemed well to do, yet sorely felt the need
of heirs, own & Sarai's seed.
Their own endeavours failing, thus Jehovah their intent.
Abimelech & Pharaoh they resolved to circumvent.
Already had they experimented upon the dusky Hagar
But did'nt like the lad, because he was half Nagar.

So they pondered well the matter, till they clearly could divine,
Thomot a royal heir, they'd got herds of sheep & kine.
Due allowance being made for the times as they were; then
General honesty prevailing, & but few left handed men;
Thus easy Abram found it, Sister for wife commented
To come it over these kings, at the expence of being cornuted. &c &c &c

Ten times the square root of a certain number together with one eighth of itself, & six — Given to find number

$10\sqrt{x}+\frac{x}{8}+6=x$ Transp &c

$10\sqrt{x}=x-\frac{x}{8}-6=\frac{7x}{8}-6$

$10\sqrt{x}=\frac{7x}{8}-6$ Clay fractions

$80\sqrt{x}=7x-48$ Sqrg both sides

Coefficient of the highest power of the unknown quantity — & adding to both sides the square of the Co. Eff of the Lower P.r

$6400x=49x^2-672x+2304$ Transp &c

$49x^2-7072x=-2304$ Compl.t square — by Multip.g Equation by 4 times the

Multip.r 196 = 4 times Co. Eff

$9604x^2-1386112x+50013184$ Ext.g Root

$98x=7072\pm7040$

$x=14112\div98=144$

Sq.r 7072 Co. Eff

$49561600(7040$

$1404\,|\,5616$

$\quad\;\;\,5616$

Again Let x^2 = equal the number — Then

$10x+\frac{x^2}{8}+6=x^2$ By Transposing & Uniting

$10x=x^2-\frac{x^2}{8}-6$ Clearing fractions & Uniting terms

$80x=7x^2-48$: or, $7x^2-80x=48$ Completing Square

$196x^2-2240+6400=1344+6400$

$196x^2-80x+6400=1344+6400$

$14x=80=\pm\sqrt{7744}$

$x=80+88\div14$

$14)\,168\,|\,12=$ Ans.

$7744(88$

$168\,|\,1344$

Religion.

What is the true & legitimate import of the term, Religion? Does it denote a tie, bond, or obligation duty & something that ought & must be done? Is it not applicable to all moral Beings, universally? and is not precisely synonymous with duty? It seems probable that its primitive meaning was more limited simple and precise, than its present, and that it was used only in a physical sense as to bind a bundle, or to tie with a cord: and that afterwards when a word was needed to denote the idea, of what men must or ought to, do as moral Beings, this word came to be used for this purpose also; and thus like many other words, through the poverty of language acquired a two fold meaning. However this may have been, is perhaps of comparatively little moment now, otherwise than as shewing how the original meaning of words may gradually & insensibly become modified & essentially perverted by popular use & acceptation. The word Religio amongst the ancient Romans, from whose language this word is obviously derived denoted certain rites & ceremonies to be observed & performed in honor of their gods: Some of these consisted in assembling together in magnificent temples, & there publicly addressing their prayers, & in singing sacred songs to the praise & glory of their gods: whilst other services consisted in taking care of the sacred chickens, watching the flight of birds, inspecting the entrails of slaughtered animals offered in sacrifice, & in interpreting prodigies & omens: all which were done by the Priests, in order to ascertain the will of the gods & thereby secure their favour. The feelings & emotions which gave birth to these rites, & which accompanied the performance of them were uniformly characterized by great gravity veneration & awe.

This was religion amongst the Romans: and amongst other contemporary nations, other observances, also purporting to be religious, were greatly diversified. But the question is, what is religion Now? assuming that the word denotes ~~exhibit~~ the idea of what men must or ought to do; it is plain that the question involves an inquiry concerning the intellectual & moral character & constitution of man: and the method of con-

84 ducting such an inquiry or investigation must obviously be very much as tho' the inquiry were "what is this mineral plant or animal good for? and what use was it intended, or is it fit, to subserve? The true answer to a question of this sort must depend on a right understanding of the properties & qualities, powers & faculties that belong respectively to these several subjects. Just so must the true answer, to the question "what is religion" depend on a right understanding of the moral constitution of man. To say - man is a moral being without having clear & distinct ideas of what is meant by the words 'good & Evil' 'right & wrong' is sheer verbiage. Now it is of the first importance to get clear & well settled ideas of these several terms. We certainly can have no definite conception of a moral Being without distinct & determinate ideas of the chief elements that make him such.

The magnetic needle is affected by iron. This we know. The motion produced by it, is purely physical & is ascribed to attraction. But what attraction is, or that power which is thus denominated, we know nothing.

In regard to moral Beings, & moral action, is not our knowledge in some degree alike circumscribed? We certainly know only that such Beings as we are, are capable of acquiring knowledge of enjoying happiness, & of suffering misery; are endowed with imaginative faculties & sundry passions, affections & emotions. These several capacities, faculties & endowments we suppose characterize a moral Being (theoretically); and also, practically when duly developed & matured by right training & education. For he, only is a moral Being strictly considered, who does only what he ought.

But this, according to the views now advocated, makes him also a religious Being: For being tied firmly, & put under bonds by his native faculties & endowments; he can perform no action at all, but in as far as he is thereby prompted; and if he do not act morally & religiously when he does what he sincerely considers right; it would seem to follow, that religion is involved in inextricable obscurity. Again motives necessarily precede action; for without them, there could be none. But what are motives? Clearly - whatever causes or induces Motion. But since the whole outer world is a mingled mass of good & Evil, & nought besides - with this qualification viz. all good if rightly used; or, all Evil, if abused. And since also

moral & religious Beings are, by their native endowments, exactly adapted to the external world, with intellectual faculties to discriminate between the good & evil, & with a correspondent faculty of being intensely affected, as they are or abuse it: (the world) It seems difficult, may, impossible, rationally to conceive that there can be any other motives to voluntary human actions, than such as arise from considerations, ~~of~~, of good & Evil either present or remote. If the attempt to teach one, blind from his birth, the art of Painting, or another, deaf, that of Music, be rightly deemed preposterous, because of the want of those faculties on which success in these arts, essentially depends: Can it be any less so, to attempt to teach Matters & things appertaining to a Spirit world, to such as are wanting in all those faculties & means, necessary to enable to test the truth & appreciate the worth, of what may be thus taught?

If human Beings are incapable of learning those arts, in which the instincts of some of the lower Animals Enable them to excel; is it not the height of absurdity to attempt to indoctrinate them, in those arts, pursuits, scenes of enjoyment & suffering, which are supposed to be opposite to an order of spiritual existences in a world, unknown?

Now let us consider, what such Beings as we are, can, must & will ought to do: for, since religion, whatsoever may be its true import must necessarily be included within this category; we shall not fail to detect that import, provided, we can clearly & satisfactorily determine what sorts of actions necessarily fall within the scope of the human faculties.

I suppose it will be conceded by all, that man is endowed with the faculty of discriminating between right & wrong: and as this conception is independent & regardless of Creeds & localities - it must be held universally true of all men of sane minds: it will be further conceded, that all men constitutionally & by nature are characteristically alike; & yet marked by individual peculiarities. Now notwithstanding this unanimity of sentiment, in regard to this discriminating faculty: there is a no less marked contrariety amongst what are considered cultivated minds, in regard to what constitutes right & wrong, & what those elements are, betwixt which the moral faculty discriminates: and here is the grand difficulty to be wrestled with; nay, the only one - religion or morality out of the question - all other human actions sink into comparative insignificance.

Beyond all question, if man were bereft of the capacity for happiness - it is inconceivable, that, tho the whole Universe should resound with songs of joy, or the howlings of despair of other beings, they would fall alike on his ear, with the same passionless indifference.

Hence it is evident that human happiness is the "primum mobile", the first & deepest principle in the human constitution, the sole end of all human effort.

Man, then, can distinguish good from evil: and he makes this distinction, not so much voluntarily, & because he chooses to do so, but rather necessarily, & because he cannot do otherwise. It does not rest with him, nor at all depend on his Will, that the various objects of sense, minister to his happiness, or, on the contrary, to his misery. The true reason or cause lies further back, & admits of no other explanation, than that such is the primitive constitution of things, the established of Order of nature. Man is provided with no other means for finding out these natural relations, nor of their availableness to happiness, than that of knowledge, derived from observation & experience: and just so far & so fast as his knowledge advances, to the same extent does he devote himself to the pursuit of what he esteems good, and to the avoidance of evil: now, thus far man acts as he MUST: Can he avoid preferring good mercurs & Scotch cabbage as articles of diet, to skunk cabbage & wild turnips - a bowl of strawberries & cream, to one of curry & rotten eggs? It is by precisely by the same means - that is - by experience that we arrive at the knowledge of right & wrong in human actions; & precisely for the same reason that we approve the one & condemn the other, that is - because of their respective tendencies to subserve, or subvert human happiness. Man in the outset, finds himself in a state of utter destitution, both of ability to do anything, & of all knowledge of the requisite for his subsistence & enjoyment: He has been unconsciously awakened into life, & introduced into a world, full of incitements to his activity, & in such order as his active faculties shall become developed - Thus far, he has been exclusively in the hands, as it were, of the Potter, & wholly subject to the guidance of invisible principles within, combined with the influence of circumstances exterior to himself: and until it can be shown how & when he became emancipated

from the government of these principles & influences; I see not why it should not be admitted, that it is continued with unabated energy thro' the whole period of life. It is not intended, by any means, to either deny or shun, the responsibility, of which all are conscious; but simply to enquire after, & to ascertain its nature & extent (taking it for granted, that religion & accountability are commensurate & coextensive) and keeping in view also, that we are now speaking of such minds only as are supposed to be ordinarily enlightened. Now what are the facts & conceded truths in every case of a wounded conscience? Is it not clear that some act has been done, or resolved on, which, at the moment, viewed under false lights, promised to be beneficial; but which, when subsequently reviewed under better lights, the Understanding disapproves? And hence the mental anguish, termed compunctious visitings of a violated conscience? And is not Repentance the natural consequence of sober second thoughts, a dispassionate review & grave reflection — the very fact of an unquiet conscience, which constitutes the chief proof of the existence of a religious faculty, necessarily involves the idea, that conscience is itself the rule & measure of religious obligation: and that whilst the conscience is at ease, the demands of religion are satisfied. But conscience is only the felt approval or disapproval of the decision of the understanding: and since, as before shown, nothing but the consideration of good & evil, either present or remote can possibly affect or in any way modify such decision; it seems to follow that the whole of responsibility is resolvable into the knowledge of good & evil as the indispensable means of attaining the chief end of human existence.

If man be as commonly supposed, wholly & absolutely dependent: How & for what can he be considered justly responsible at all, especially to that Power on which he is dependent? Does not the idea of responsibility involve that of absolute freedom? the ideas of absolute dependence & accountability who can reconcile? Is it possible for us to conceive of a Potter, who should destroy a gallon jug, that he had made, that it should overtask itself to the capacity of a barrel — and that he should dash it to pieces for its disobedience? Is it at all more conceivable, that a sensitive & intellectual piece of Machinery can enlarge, diminish, or in any other way modify its powers & capacities, than can that of a jug? It is obvious, that the idea here involved is none other, than that of self-creation: For to suppose that

a created Being exercising functions, other than such as properly belonged to, & were within the scope of its primitive endowment, is to suppose him acting thus in spite of constitutional restrictions, independently & on his "own hook". But to avoid further entanglement in the meshes of liberty & necessity, let us return to the question of responsibility, regarded in such light as renders it at all intelligible: And here is it not obvious that the power or principle, to which we are responsible, is within, if not a part of ourselves? Is not the whole process of wrong-doing, or reflecting on it, of penitence therefor, of carefully guarding against a repetition of the evil act, & of mental quiet & security that succeed, wholly & exclusively limited to ourselves? We are certainly not cognizant that any Being exterior to ourselves, has any thing to do with any part of this process: And inasmuch as this process is to be met with amongst all sorts of men, irrespective of their superstitious creeds, & as well amongst those who worship the Devil as those who worship God; we seem compelled to the conclusion, that the whole theory of accountability is resolvable into the subtle & mysterious operations of our own minds. The ideas here suggested must not be understood either to exclude, or to detract from the doctrine of the Divine Agency in the absolute government of the Universe.

The constitution of the world as we find it, & the general Providence, or order of Nature by which it is sustained are considered as affording us all the light we possess, & all that is needful to enable us to live in such a manner, as to ensure the greatest glory to God, & the greatest amount of happiness to man: This last however is that with which we are at present concerned — the only legitimate end of all scientific research. The following unquestioned facts, & considerations inseparable from them, seem to me to have a direct bearing upon the proposition here stated: and to be conclusive as to its truth. The first of these facts, is the actual & real existence of a Code of Laws (supposed for the most part, to have God for their Author; & by so much the better if the supposition be true) called "the Laws of nature, which are constant & immutable in their operation in their operation & universal in their scope & application: Whose irresistible energy and authority are embodied in principles, which so pervade & vivify all forms & conditions of being, that no event, great or small, does or can take

place, but by & thro' their Agencies. Now the first prominent consideration, inseparable from this code, is, that the whole circle of modern science is exclusively based upon its real existence & supreme energy: and so much so, that no event NOW transpires, worthy the attention of thinking minds, which is not spontaneously regarded as an inevitable sequence in the Order of nature: and such is the degree of confidence among such minds, that whither the event can be traced to, or clearly connected with its supposed antecedent or not, it matters not, it is thus regarded as a natural event; and that the principle that gave birth to it, as having thus far, eluded observation. Such amongst the yet unexplained occurrencies, may be reckoned the "Aurora Borealis", aerolites polarity of the Magnetic Needle, the rotatory movement of a speck of camphor, dropped into a basin of water, & its immediate state of quiescence by means of a single drop of any aromatic fluid &c &c &c saw all the rest. It is that of the moral & intellectual constitution of man, viewed in connexion with the constitution of things. Here we find ourselves introduced into a world, teeming with good & Evil, promiscuously intermingled; & subject to no prohibition as to appropriation & enjoyment, save that of knowledge, previously to be elaborated by diligent of Observation & experience. We find herein precisely such incitements to activity, as are requisite to awaken & call forth our dormant mental powers, without which we should remain at best, but doubtful competitors for precedence with the Ourang-Outang. Seeing that such Beings as we are, can act at all, only by the impulse of motives, we are here supplied with a key to their arcana, the whole machinery of human actions is hereby revealed. The Desire of Happiness incorporated, or provided for in the constitution of Man; and the KNOWLEDGE of Good & Evil, the indispensable MEANS of gratifying that desire. Thus much may be regarded as clearly indicated by the established Order of Nature.

A brief recapitulation of the foregoing (Tho' it be confused disultory & ill arranged) facts, it is hoped, will render more perspicuous our idea in what true religion consists — and in the first place, the idea of the tie or bond, supposed to be implied by the force of the term, is to be found ONLY in the stringent and unyielding force of the laws of Nature. and in the next place

These laws as arbitrarily impel us to seek after good as a means of

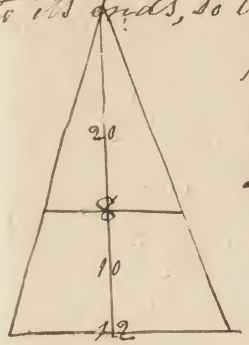
 happiness, as they do, inert matter, to obey the force of gravitation; or a lower animal or vegetable, its instincts.

We enjoy or possess perfect physical liberty, within the limits prescribed to our physical powers, to do whatsoever we will; & yet we are absolutely constrained to will whatsoever shall appear to our understandings, at the moment of willing (all things considered, particularly present & remote enjoyment) as the preferable good.

The order of Nature or Providence affords abundant means for promoting human happiness; the chief of which is knowledge. Without knowledge, all other means are as likely to be converted into curses as blessings. The incentives to knowledge, are desires & wants.

True Religion enjoins obedience to the laws & institutions of nature; and implies such provision of their operation, as to enable us in some degree to modify & controul, & direct it. (operation)

A Frustum of a Cone is ten feet in length, & its bases 12 feet & 8 feet in diameter. Where shall it be cut off by a section parallel to its ends, so that the parts shall be equal to each other?



$12^2 \times .7854 \times 10 = 1130.9760 - 335.1040 \overset{2)}{=} 795.8720 = 397.9360$

$8^2 \times .7854 \times 6\frac{2}{3} = 335.1040 + 397.9360 = 733.0400$

$335.1040 : 733.0400 :: 20^3 : 17500$ the cube root of which is 25.962744

25.9627447 which subtracted from 30

leaves 4.0372553 & multiplied by 12

48.4470636 subtracted from 120.

Answr in Inches 71.5529364

Two Men buy 80 lbs of Beef at 4 cents a pound $3.20 cents On dividing it between them, One takes 50 lbs, & the other 30 lbs: But it is considered that the larger quantity is worth ½ cent a pound more than the smaller. How much shall each pay?

Suppose the better part worth 4¼ cents × 50 lbs = 212½

3¾ × 30 = 112½

3.25

3.20

1st Error +5

1st Sup 4¼

5

21 ¼ $\frac{2}{8}$

2d Sup. 4⅛

5

20 5/8

Sum of Errors 10) 41 7/8 sum of the products

Prices { 4 $\frac{3}{16}$, 3 $\frac{11}{16}$ } $\frac{15}{8} \div 10 = \frac{15}{80}$ or $\frac{3}{16}$

2d Sup

4⅛ × 50 = 206¼

3⅝ × 30 = 108¾

315

320

2d Error −5

4 3/16 = $\frac{67}{16}$ × 50 = $\frac{3350}{16}$

3 11/16 = $\frac{59}{16}$ × 30 = $\frac{1770}{16}$

Sum: 5120

16) 5120

3.20 Proof

126} a plank 12 feet long – 12 inches wide at one end & 8 inches at the other, is required to be divided into 2 equal parts, by a line parallel to its ends – what is the length of each section – and the width of the plant at the point of severance?

It is easily found that if the plank were divided midway from its ends – one part would contain 5.5 square feet, & the other 4.5. Hence it is conceived that this ratio between the areas, must be the ratio between the lengths of the sections inversely. Thus – putting x – for the longer section, & $12 - x$ for the shorter; we have this proportion $x : 12 - x :: 5.5 : 4.5$

$$4.5x = 66.0 - 5.5x$$
$$\frac{5.5x}{10.0x} = 66. \text{ and } x = 6.6 = 79.2 \text{ inches – the longer}$$
$$\& \ 64.8 \text{ Do – the shorter Sectn}$$

The number of square inches in the plank is 1440. The ½ of which 720, divided by 79.2 & 64.8 – give severally the average {in each case widths of 9.09090909 & 11.111111 – 72 remainder –

av. width remdr

79.2) 720.00000 9.090909 – 72 {11.11111) 720.00000 (64.8 – 72

7128
7200
7128
7200
7128
7200
7128
72

9.09090909
79.2
1818181818
8181818181
6363636363
719999999928
72 remdr
720.000000000

5333340
4444444
8888960
8888888
72

64.8) 720.000 (av. width of Sectn 11.111111 – 72 remdr

648
720
648
720
648
720
648
720
648
72

1111111111 – 72
648
8888888960
4444444444
6666666666
720.000000000

Answr – 79.2 inches & 64.8 ditto

Religion.

What is the true & legitimate import of the term "religion"? Does it denote a tie, bond, obligation or duty — something that ought, that must be done? Is it not applicable universally to all moral Beings: and is it not synonymous with duty? It seems probable that its primitive meaning was more limited, simple & precise, than its present, and that it was employed only in a physical sense — as, to bind a bundle, or to tie with a chord &c as that afterwards, when a word was needed to denote the idea of what men must or ought to do, this word came to be used for this purpose also, & like many other words thro' the poverty of language, acquired a double meaning. However this may have been, perhaps, is of comparatively little moment now, otherwise than as shewing how the original meaning of words may gradually & insensibly become modified & essentially perverted by popular use & acceptation.

The word 'religio' amongst the Ancient Romans, from whose language this term is obviously borrowed, denoted certain rites & ceremonies to be observed & performed in honor of their gods: Some of which consisted in assembling together in splendid temples, in addressing their prayers, & in singing hymns to them (their gods); whilst other services consisted in taking care of the sacred chickens, watching the flight of birds, inspecting the entrails of slaughtered animals offered in sacrifice & in interpreting prodigies & omens; all which were done by the Priests, in order to ascertain the will of the gods & to secure their favour. The feelings & emotions which gave birth to those rites, & which accompanied their performance were characterised by gravity, fear reverence, veneration & awe.

This was religion amongst the Romans. Amongst other contemporary nations, other rites ceremonies & observances, also purporting to be religious, were greatly diversified.

But the question is, what is, & what constitutes, religion now? Assuming that it is a term, used to denote the idea of what men must, or ought to, do; it is plain that the question or inquiry involves concerning the intellectual & moral character & constitution of man: What he can, must, ought to, & will do, considered as a sensitive moral & intellectual Being. And the method

of conducting such an investigation must, obviously be very much as tho' the inquiry were — what is this, or that mineral or plant good for? or what use was it intended, or, is fit to subserve? Or, what can & will this or that kind of lower animal, do? and to what useful purposes can they be made subservient? The true answer to any such questions must depend on a right understanding of the properties & qualities, powers & faculties, that belong respectively to these several subjects. In fact so, must the true answer to the question — What is religion? depend on a right understanding of the moral constitution of man.

To say — man is a moral Being — without having clear & distinct ideas of what is meant by the words "good & evil, right & wrong" is mere verbiage. Hence, it is of the first importance to get clear & well settled ideas of these several terms. We certainly can have no definite conception of a moral Being without distinct & determinate ideas of the chief elements, that make him such. The magnetic needle is affected by iron. This we know. The motion produced is purely physical, & is designated by the word attraction. But what attraction is, or that power which is thus denominated, we know nothing. This is the limit of our knowledge; and our reason does not enable us to go beyond it.

In regard to a moral being; is not our knowledge in some degree alike circumscribed? We certainly know only that beings like ourselves are capable of knowledge, & happiness & misery. Beyond this (our capacities for knowledge & happiness) can we, do we know any thing of ourselves? In what these capacities consist, we are as much in the dark, as we are in regard to gravitation & electricity. In addition to these capacities, we may add — that we are endowed with powers of imagination, with passions affections, and sundry emotions. These several capacities, we suppose, characterize a moral Being: Or, rather, are essential to such a being; and really constitute a moral Being, only, when duly educated & developed. For he only, who does what he ought, can justly be regarded as moral or religious. To draw forth, to nourish, to invigorate and to train these several faculties, to a state of healthful maturity constitute the sole purpose, the only end of education.

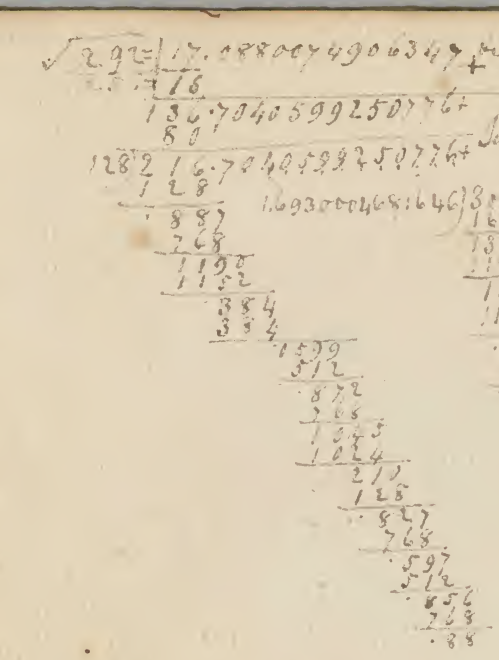
If the attempt to teach One, blind from his birth, the art of painting; or another, deaf, that of music, be rightly deemed preposterous, because of the want of those faculties on which success in these arts depends; can it be any less so, to attempt to teach matters & things, entertaining

to a spirit world, to such as are wanting in all those faculties, necessary to enable them to test the truth, & to appreciate the worth of what may be there taught? If human Beings are incapable of learning these arts, in which the instincts of the lower animals enable them to excel, is it not the height of absurdity to attempt to indoctrinate them in those other arts, pursuits, scenes of enjoyment or suffering, which are supposed to be applicable to an order of spiritual existence in a world unknown?

Let us then proceed to consider what men, endowed as they are, can, will, & ought to, do: for since religion, whatsoever may be its true import, must necessarily be included within this category; we shall not fail to detect that import, provided we can clearly & satisfactorily determine what actions necessarily fall within the scope of the human faculties. However small or little soever our moral & religious conduct may be affected by the action of the heart & lungs; & those movements & continued transformations in the material parts of our bodies, which properly belong to the vital economy; I shall omit for the present, to notice — because they are powers manifestly distinct from, & independent of ourselves & therefore not within the purview of the present inquiry:

It is conceded by all, that man is endowed with the faculty of discriminating between right & wrong: And this conception is wholly independent & regardless of Creeds & Localities. It may hence be regarded as true of all men universally; that is — of all men of sane minds. It is also conceded that all men are constitutionally & by nature, alike in their general characteristics; tho' marked by individual peculiarities. and yet notwithstanding this unanimity of sentiment in regard to the discriminating faculty; there is a no less marked contrariety amongst what are considered cultivated minds, in regard to what constitutes right & wrong & what those elements, are, between which the moral faculty discriminates.

Nor is there any other conceivable way of adjusting this conflict of opinions, than by such means as are derivable from the knowledge of Good & Evil? If not: then it would seem, that good & evil are the primary & fundamental principles which equally & alike constitute, the foundation of right & wrong, of morals, of duty of religion & of human happiness: For if the capacity for happiness & misery be supposed to be abstracted from man; it is utterly inconceivable, that there could be aught left, to tie or attach him to existence: So that, whether the Universe resounded with the songs of joy, or, with the howlings of despair; they would fall upon his ear with the same passionless indifference.) See page 128.



$\sqrt{292} = 17.0880074906347 + \frac{5}{8} \times 128$ Dec.

$\begin{array}{r} 16 \\ \hline 136.7040599250776 + \\ 80 \\ \hline 128\,216.7040599250776 + \\ 128 \\ \hline 887 \\ 768 \\ \hline 1190 \\ 1152 \\ \hline 384 \\ 384 \\ \hline 1599 \\ 512 \\ \hline 872 \\ 768 \\ \hline 1045 \\ 1024 \\ \hline 210 \\ 128 \\ \hline 827 \\ 768 \\ \hline 597 \\ 512 \\ \hline 856 \\ 768 \\ \hline 88 \end{array}$

Sum 1.6930004681646 = Y nearly.

$1.6930004681646 \,)\, 300.0000000000000000 \,(\, 177.200$

$\begin{array}{r} 1693000468164 6 \\ \hline 130699953183540 \\ 118510032771522 \\ \hline 121899204120180 \\ 118510032771522 \\ \hline 338917134865 80 \\ 338600093632 92 \\ \hline 317041232880 0 \end{array}$

One Lang asked how many guineas he had? answered, that if the number were added to the square root of four times the number, the sum would be 440.

Let x represent the number

then $\sqrt{4x} + x = 440$

and $\sqrt{4x} = 440 - x$ sqr. both sides

$4x = x^2 - 880x + 193600$

Comp. sq. & transp.

$x^2 - 884x + 195364 = -193600 + 195304$

$x = 442 \pm \sqrt{-193600 + 195304}$

$+42$

$400 = x$

$\begin{array}{r} 193600 \\ \hline 1764 \,|\, 42 \div 442 = 400 \text{ the number req.} \\ 10 \\ \hline 82 \,|\, 164 \\ 164 \end{array}$

$\begin{array}{l} 440 - x \\ 440 - x \\ \hline 176 - 440x \\ 176 - 440x + x^2 \\ \hline 193600 - 880x + x^2 \end{array}$

Sq. & Coefficient

$\begin{array}{r} 442 \\ 442 \\ \hline 884 \\ 1768 \\ 1768 \\ \hline 195364 \end{array}$

The sum of any two numbers (whose diff. is one) is equal to the diff. of their squares and the sum of any two numbers, multiplied by their diff. is equal to the difference of their squares.

Twice the sum of the squares of any two numbers exceeds the square of their sum, by the square of their difference.

A & B own 177 Acres of land in 2 Lots Lot A containing 3300 Rods — B's containing 1693 Rods. Upon dividing it was found that the last Rood was allotted & it was worth 1 Dollar more per Acre than that allotted to B. How much is each Rood?

Let x equal B's last & it's Price per Acre then $y = 300 - x$: $y = 300 - x = \frac{300}{4}$

300 Dolls — A's last will be 300 Dolls — x

Substitute & value of x —

$$(300 - \frac{300}{4}x)(4 + \frac{3}{4}) = 300$$

$$300 - 300 + 900 - \frac{900}{4} = 300 \quad \text{Transp}$$

$$300 - \frac{900}{4} = 300 + 300 - 225 = 375$$

$$3) \quad 300 - 225 = 375$$

$$25) \quad 100 - 75 = 125$$

$$4 - 3 = 5$$

$$4^2 - 3 = 5^2$$

$$4^2 - 5^2 = 3$$

$$x^2 - 5x = 3 = y^2 - 5y + \frac{25}{64} = \frac{3}{4} + \frac{25}{64}$$

$$y = \frac{5}{8} \pm \sqrt{\frac{3}{4} + \frac{25}{64}} = \sqrt{\frac{192}{256}} + \frac{100}{256} = \sqrt{\frac{292}{256}}$$

Let the last Lot

Rods Land — B's Land = 1693 Rods = 1693 Dolls Price
1693 Rods B's Land = 3300 Rods = 1.693 Dolls Price
1693 1128 A's Land = 3300 Rods = 2.443 Dolls Price
13079 .750 Diff Price
11855
12190
11855
3350
3384 — 1.693 Price
44 — 5328 — 4 added

15948	122.8
12432	2.443
1772	3684
53.30.000	4912
	4912
	2456
	53.0.0.1.34

A's Land 122.8 @ 2.443 = 300.0004
B's — 177.2 - 1.693 = 299.9996
.7508

Ratio 307 & 443

17.088007 49 + 5/8
14
8
13 6.7040599 2
21 6.7040599 2
126 216.7040599 1 (1.693004 68 = X
126
88
76
1130
1132
384
384
00 00599
512
8.72
7.68
1840
1536
16

292 | 17.088007 43 (16
256
26 1156
6 1156

27 1192
7 1189
3 4930
2 7264
34 2166 | 2736 00
27 3344
34.7602 50000

17 + 5
16 8
136 128
8

128 216 (1.69
125
880
768
1120
1152
48

17.088 = 5
16
136.704
80
126 | 216.704 | 1.698 = y Rod
128
88
76
1192
1152
384
384

34.1760 07 | 256 00 00 00
23 9232 049
10 2527 95 100
34176014 4 | 13 6704 0572
3 0 0 3 7 5 4 = 2400
34.1760158 9 | 3075 4 = 301
9
2.1760 3809 00

Problems — by Thomas Barlow 2d — June 13th 1829

1.

If the square root of any number be multiplied by 2.3.4 &c. the product will be equal to the square root of ~~the product of~~ said number multiplied by the square of said multiplier — —

Ex: Let 100 be the number & 10 its square root multiplied by — 4
40 which is equal to the ~~square~~ root of the product of $100 \times 16 = \sqrt{1600} = 40$ —

Prob. 2d By Division — same — reversed —

Prob. 3d

If any number whose square root is involved with itself is required — Rule — Take the square root by subj last quot. fig &c

Ex. Let the square root of a number added to itself be 56 — what is the numr? $56 \mid 7 \times 7 = 49$ answer
49
7

Prob. 4 —

(added to)

If any number (with) the square root of 2.3.4 &c times its [product] number, be divided by 2.3.4 &c the quotient will be the number with its root — which root (extracted) multiplied by 2.3.4 &c will give the root ~~required~~ — or number required — —

Ex. A man being asked how many Dollars he had in his Pocket — and if the number be multiplied by 4 — & the square root of the product be added to the number it will make 440 —

Let 440 be divided by 4
1 10 | 10 greatest sqr & 10 remdr —
1
20 | 10
4
40 root added to 400 = 440
& 400 the number

$\sqrt{4x} = 440 - x$ } Squa
$440 - x$
$176 - 440x$
$176 - 440x$
$4x = 193600 - 880x + x^2$
$\sqrt{4x} + x = 440$ $x^2 - 884x = -193600$
$4x^2 - 3536x + 781.456 = -774400 + 781.456$
$2x = 884 + -\sqrt{7056} \mid 84$
84
$x = 80.0 \div 2$ $164 \mid 656$
656
$x = 400$

774400
root
7056 | 84
64
164 | 656
4 | 656

The Number

Prob. 5.th

When any Number with 2.3.4 &c times its square root is given to find its root or Number — Rule

Divide the given Number by the square of the 2.3.4. &c by which it has been increased & the quotient will be the number with its root involved, which root, multiplied by said square will give the root required

Ex. 1.st

Prob. 6.th

To form a Square within any given number, that shall be equal to ½.⅓.¼. &c of the Remainder — Rule

Multiply the whole Number by 2.3.4. &c then extract the square root, by adding the last quotient fig. &c (by Prob. 3.d) which root divided by said Multiplier gives the square required.

The product of the sum & difference of any two numbers, is equal to the difference of their squares. And if such numbers have a difference of but one

~~The sum of any two numbers multiplied by their difference is equal to the difference of their squares.~~

the difference of their squares will be equal to their sum: if of two — of twice their sum: if of 3: to 3 times & so on &c. Aug. 6. 1837

Twice the sum of the squares of any two numbers, exceeds the square of their sum — by the square of their difference

The sum of the squares of any two numbers exceeds double their product by the square of their difference. Ante Mas. admit

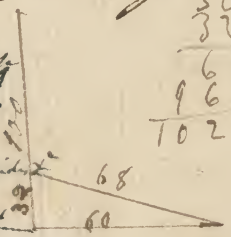
The square of the sum of the legs of any right angled triangle, exceeds the square of the hypothenuse, by four times the area of the triangle

A tree one Hundred feet high stands by the side of a Stream Sixty feet wide — How far from its top must it be cut off so that it will just reach the opposite bank?

Rule.

Divide ½ the sum of the Squares (of the height of the tree, & the breadth of the Stream) by the Height of the tree — Or, the whole sum, by twice the Height — the Quotient will be the Answer. The difference between the ½ sum &c. and the square of the height of the tree, divided by the height, will give the length of the stump. By Algebra

Let x = Dist from top to the top of stump
then x being the Hyp. of a right ang. Δ
x^2 = the squares of the 2 sides — viz
$60 + 100 - x$ — this last expanded gives
$10000 - 200x + x^2$. & 60^2 being 3600 —
$x^2 = 13600 - 200x + x^2$. Expunging on both sides x^2
& transposing $-200x$
$200x = 13600$
& $x = 13600 \div 200 = 68$ the Hypot.



32	60	68
32	60	68
64		544
96	3600	408
1024	1024	4624
	4624	4624

Theorem.

The Hypothenuse in any right angle triangle is equal to the square of its base, together with the square of the sum of its other leg & ~~hypothenuse~~ itself, divided by twice said sum. Or, It is equal to half the sum of the squares, of the base, & of its other leg together with itself, divided by the sum of said other leg & itself.

2d

Half the sum of any two squares, divided by a side of the greater, ~~is~~ is equal to the Hypotenuse of a right angle triangle, whose base is a side of the less.

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Paid. Daughtry for Sand Lap: & Sand ——————— } $0.43.